

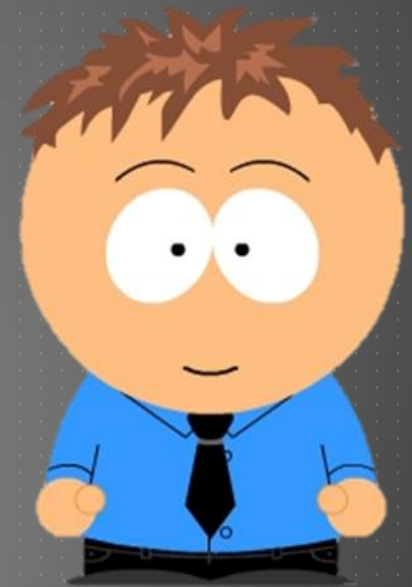
DIFFERENTIATION AND INTEGRATION PART 4

Mr C's IB Standard Notes

In this PDF you can find the following:

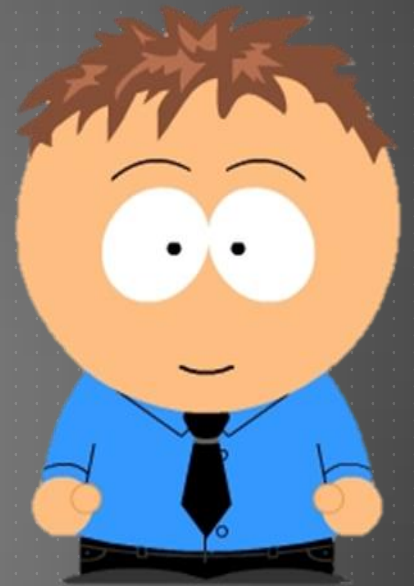
1. [Kinematics](#)
2. [Worked Example](#)

Make sure you read through everything and then try examples for yourself before looking at the solutions



KINEMATICS

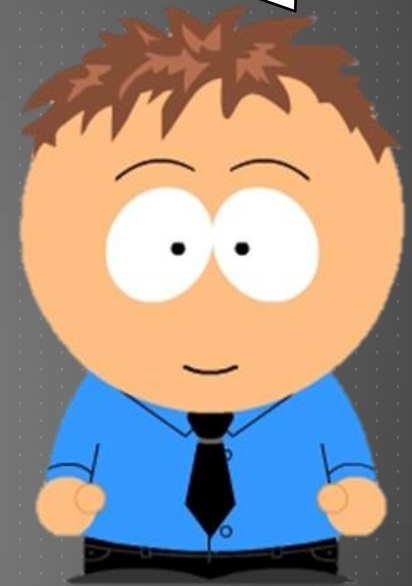
Working with displacement, velocity and acceleration



KINEMATICS

Working with displacement, velocity and acceleration

One very useful application of
calculus is in the topic of
kinematics

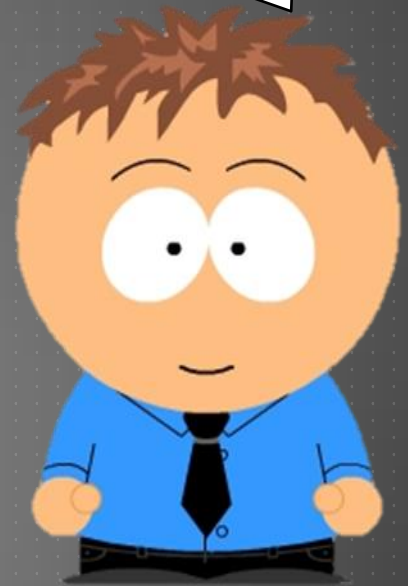


KINEMATICS

Working with displacement, velocity and acceleration

Most people know the
relationship

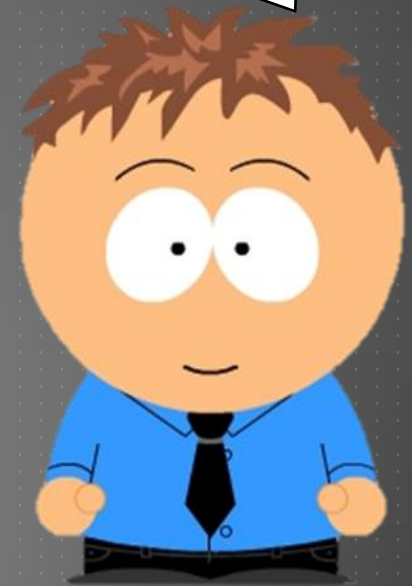
$$speed = \frac{distance}{time}$$



KINEMATICS

Working with displacement, velocity and acceleration

When working with vector quantities this becomes
$$velocity = \frac{displacement}{time}$$



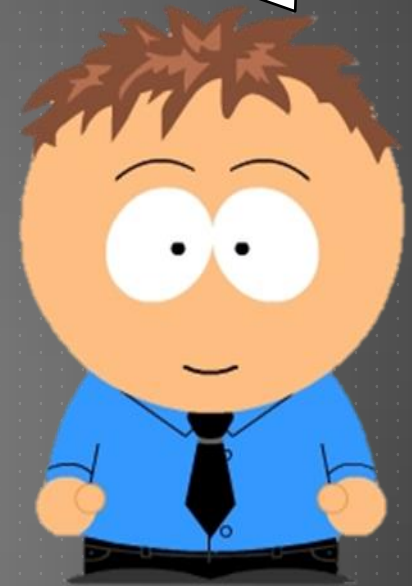
KINEMATICS

Working with displacement, velocity and acceleration

Displacement = s , velocity = v , Acceleration = a

Time = t

When working with vector quantities this becomes
$$velocity = \frac{displacement}{time}$$



KINEMATICS

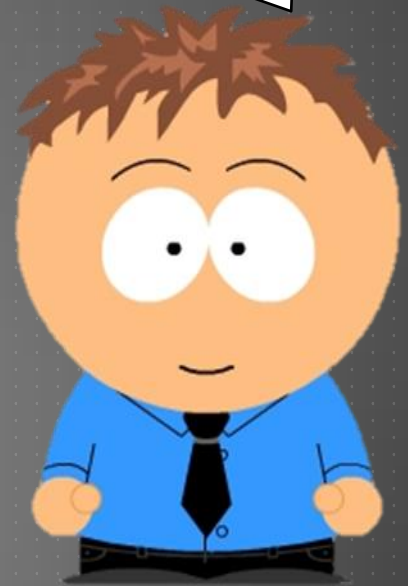
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$$v = \frac{s}{t}$$

When working with vector quantities this becomes
 $velocity = \frac{\text{displacement}}{\text{time}}$



KINEMATICS

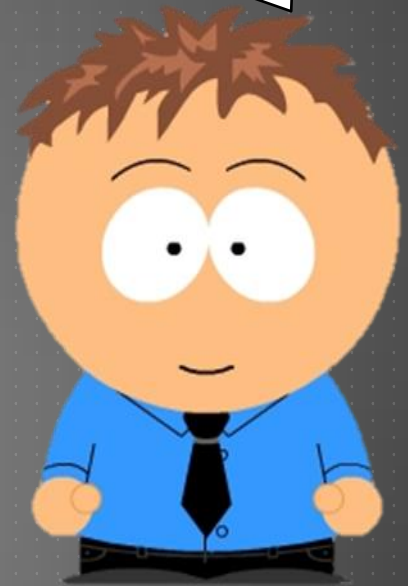
Working with displacement, velocity and acceleration

Displacement = s , velocity = v , Acceleration = a

Time = t

$$v = \frac{s}{t}$$

More precisely it can be stated as
velocity = $\frac{\text{change in distance}}{\text{change in time}}$



KINEMATICS

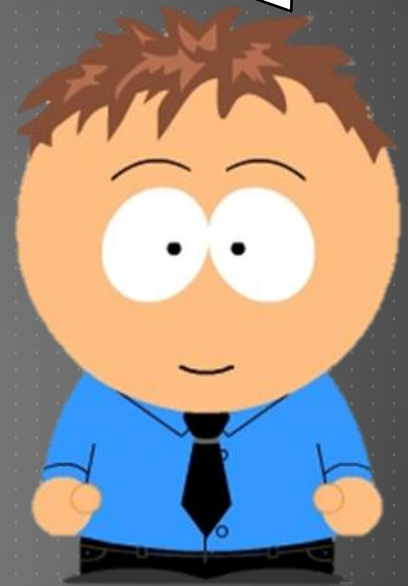
Working with displacement, velocity and acceleration

Displacement = s , velocity = v , Acceleration = a

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$$v = \frac{\Delta s}{\Delta t}$$

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KINEMATICS

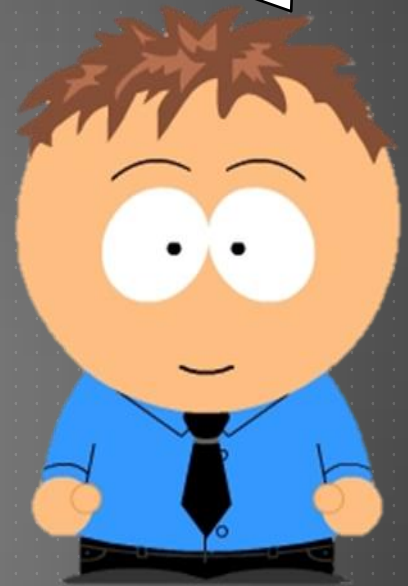
Working with displacement, velocity and acceleration

Displacement = s , velocity = v , Acceleration = a

Time = t

$$v = \frac{\Delta s}{\Delta t}$$

Therefore differentiating distance
with respect to time gives
velocity



KINEMATICS

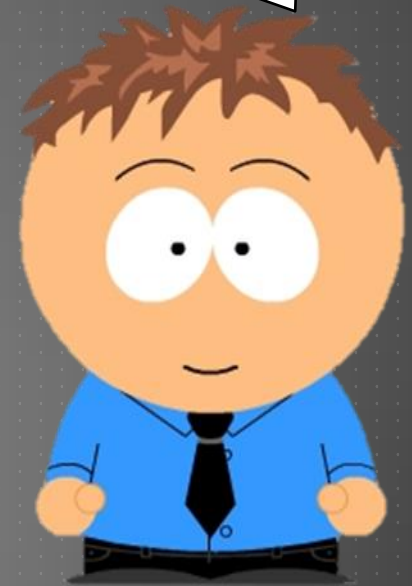
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KINEMATICS

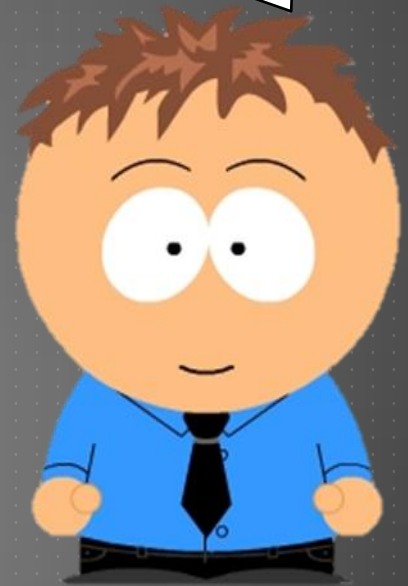
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Similarly if you differentiate velocity you get acceleration



KINEMATICS

Working with displacement, velocity and acceleration

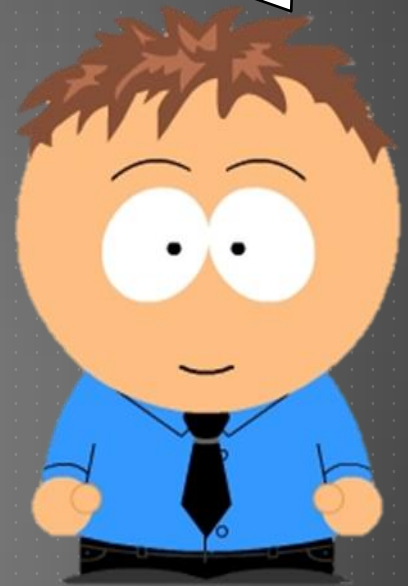
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$$v = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Similarly if you differentiate velocity you get acceleration



KINEMATICS

Working with displacement, velocity and acceleration

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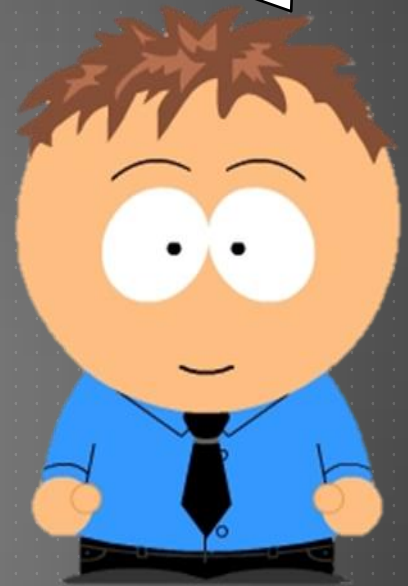
Time = t

s

$$v = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

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It is important that you remember how to convert between s , v and a



KINEMATICS

Working with displacement, velocity and acceleration

Displacement = s , velocity = v , Acceleration = a

Time = t

differentiate

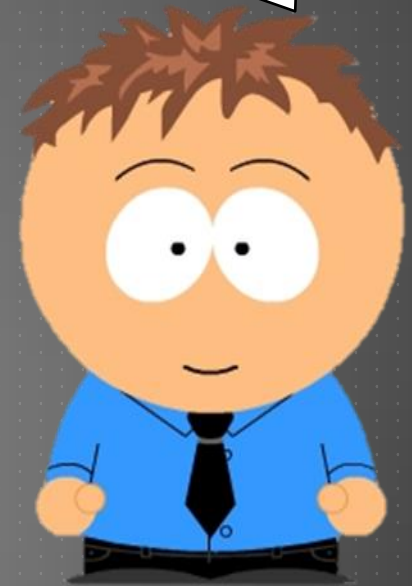


s

$$v = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

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Differentiate from distance to velocity, and again to acceleration



KINEMATICS

Working with displacement, velocity and acceleration

Displacement = s , velocity = v , Acceleration = a

Time = t

differentiate



s

$$v = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

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integrate



Or Integrate from acceleration to velocity and then distance



EXAMPLE

I.

Amy does a bungee jump from a platform 50m above a river. Let h be her height above the river, in metres, at a time t seconds after jumping.

Her velocity is given by $v = 2t^2 - 10t$

a) What is the initial acceleration Amy experiences?

b) At what time is Amy's velocity zero?

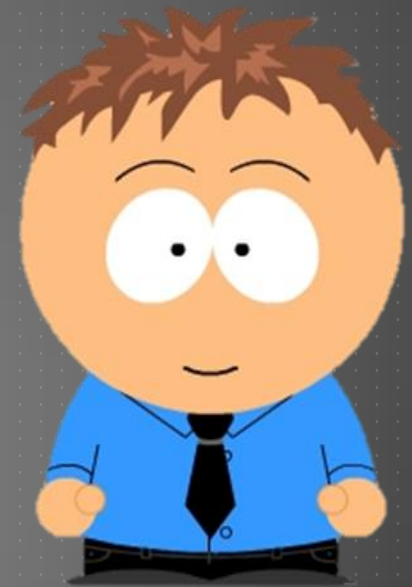
c) How close to the river does Amy get?

SOLUTION

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SOLUTION

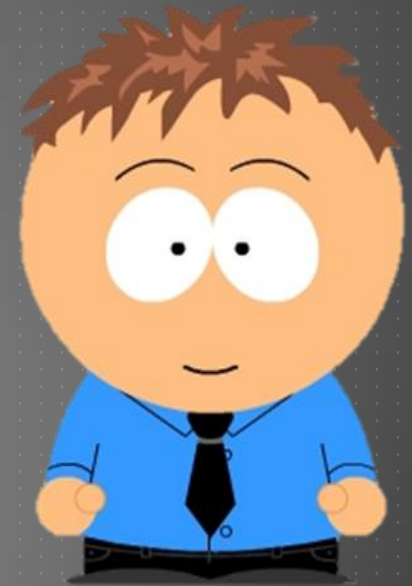
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We have a formula for *velocity*, but are asked to find *acceleration*. We must therefore differentiate



SOLUTION

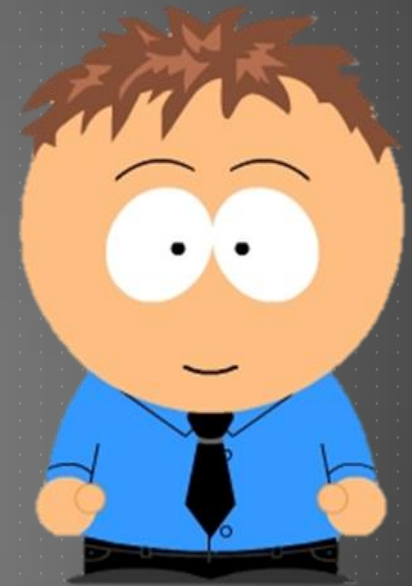
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SOLUTIONS

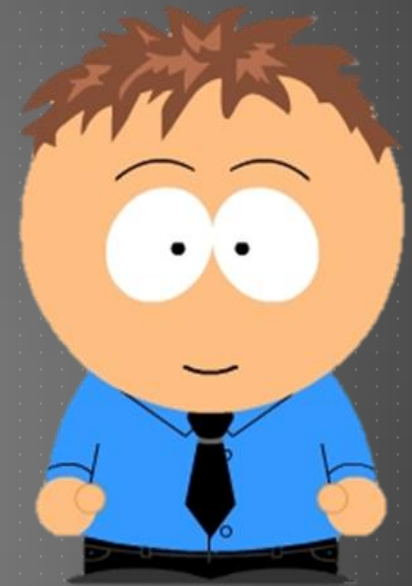
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$$v = 2t^2 - 10t$$

$$a = \frac{dv}{dt} = 4t - 10$$

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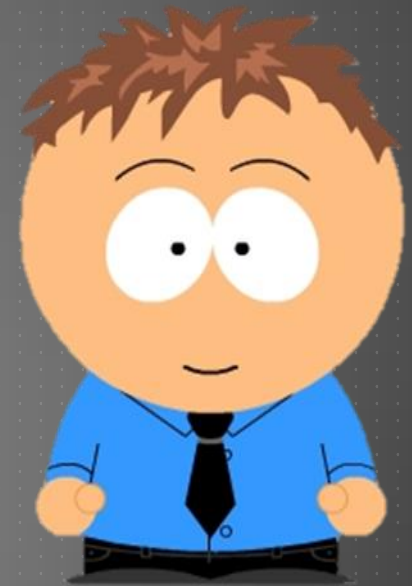
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The **initial acceleration** occurs at the start, i.e. when **$t=0$**



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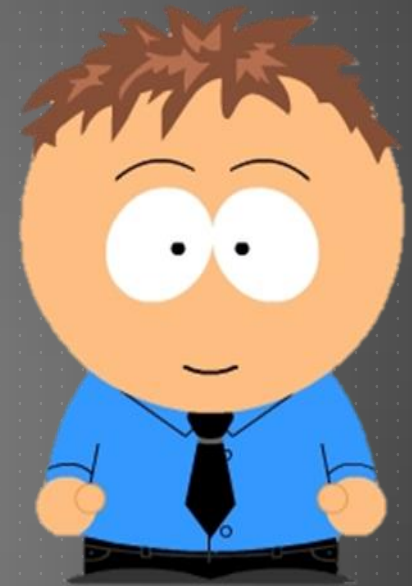
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The negative value means that the acceleration is in a downwards direction (as expected)



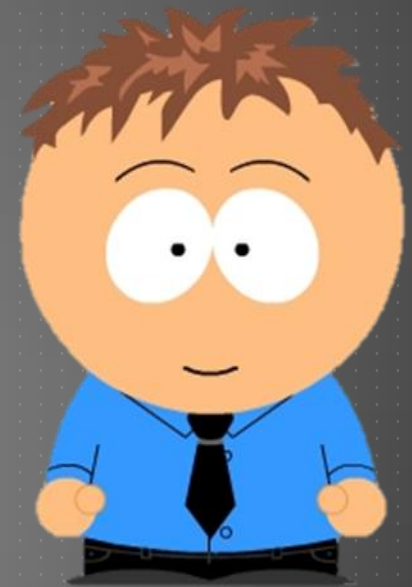
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b) At what time is Amy's velocity zero?

$$v = 2t^2 - 10t$$



SOLUTION

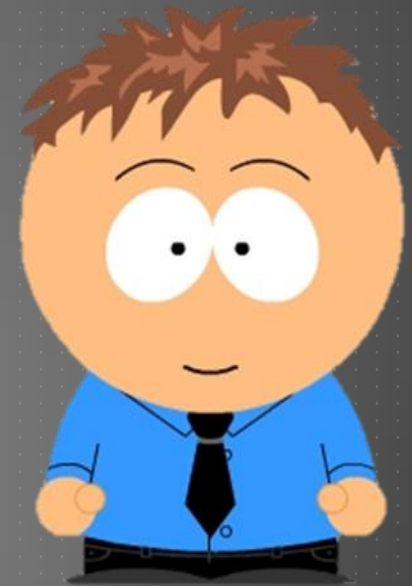
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We need to find when $v = 0$.
That's fairly straight forward.



SOLUTION

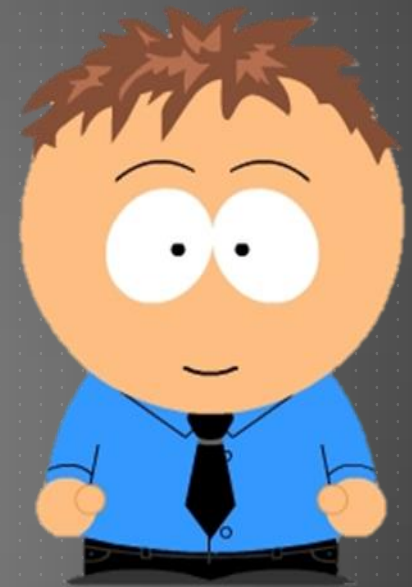
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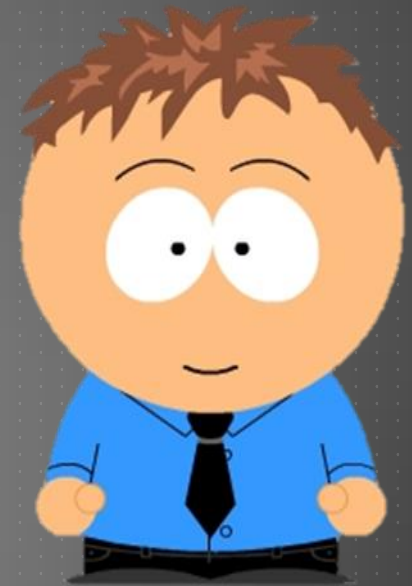
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Solve the quadratic using your preferred method



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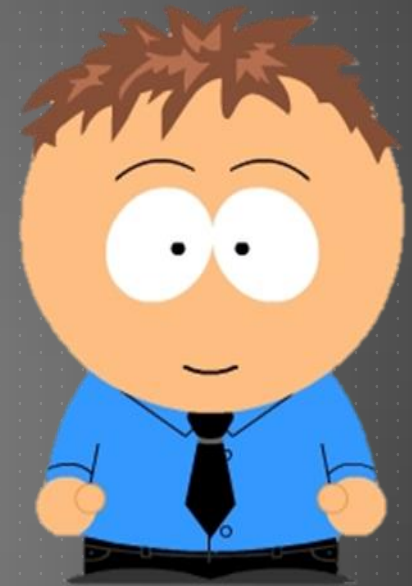
b) At what time is Amy's velocity zero?

$$v = 2t^2 - 10t$$

$$0 = 2t^2 - 10t$$

$$t = 0 \text{ or } t = 5$$

The quadratic should give two solutions



SOLUTION

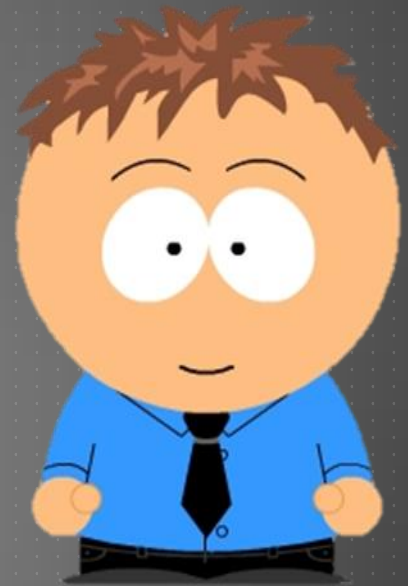
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c) How close to the river does Amy get?

$$v = 2t^2 - 10t$$

$$v = 0 \text{ when } t = 0 \text{ or } t = 5$$



SOLUTION

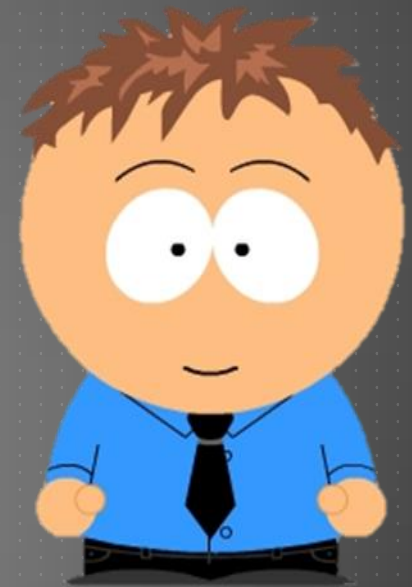
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The bungee has reached its minimum height when the velocity is 0 (as it changes from -downwards to +upwards)



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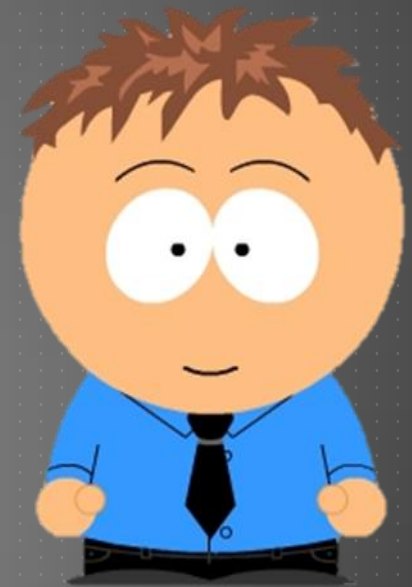
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We know this takes 5 seconds.



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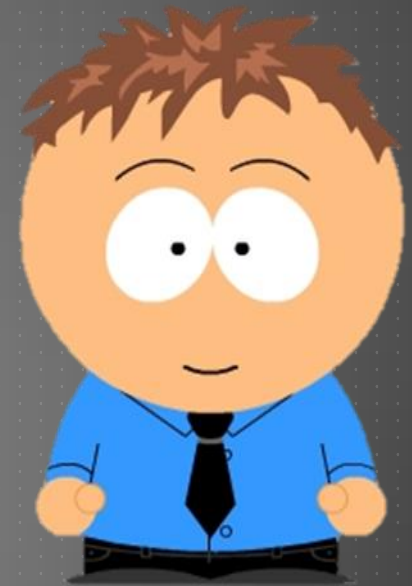
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So we must calculate how far is travelled in those 5 seconds



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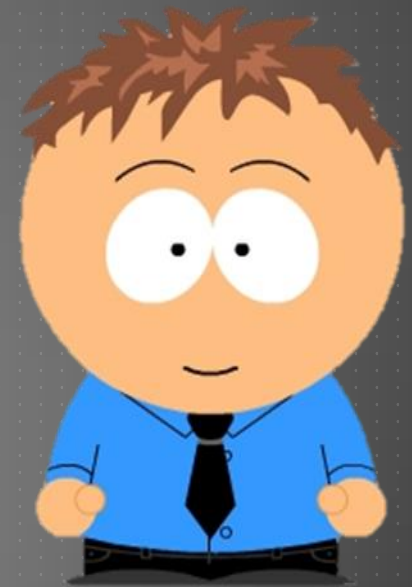
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$$v = 2t^2 - 10t$$

$$v = 0 \text{ when } t = 0 \text{ or } t = 5$$

$$s = \int v \, dt$$

So we must calculate how far is travelled in those 5 seconds



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$$v = 2t^2 - 10t$$

$$v = 0 \text{ when } t = 0 \text{ or } t = 5$$

$$s = \int v \, dt$$

$$\int_0^5 2t^2 - 10t \, dt$$

So we must calculate how far is travelled in those 5 seconds



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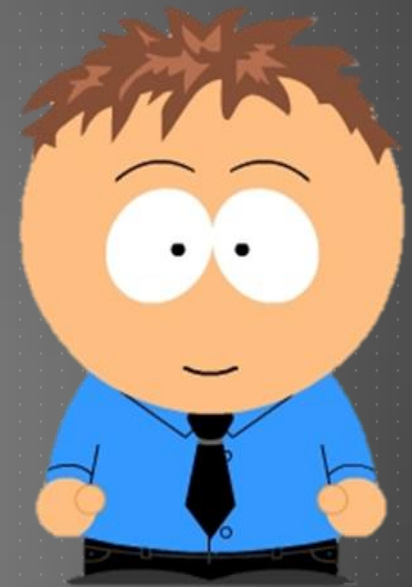
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$$\int_0^5 2t^2 - 10t \, dt = \left[\frac{2}{3}t^3 - 5t^2 \right]_0^5$$

So we must calculate how far is travelled in those 5 seconds



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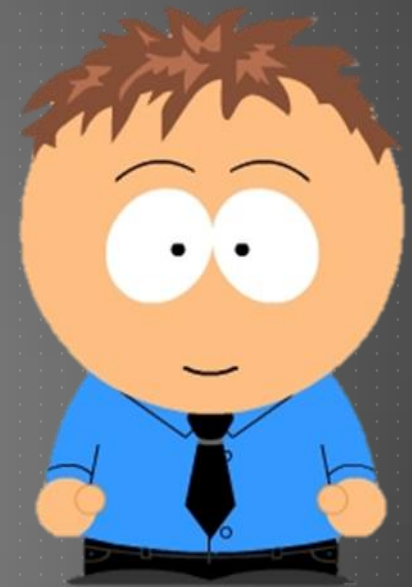
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$$= -41\frac{2}{3} \text{ m}$$

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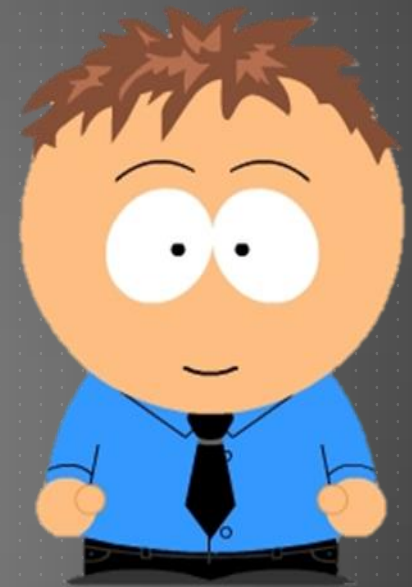
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The platform is 50m, she falls $41\frac{2}{3}$ m, therefore....



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$$50 - 41\frac{2}{3} = 8\frac{1}{3} \text{ m from the river}$$

