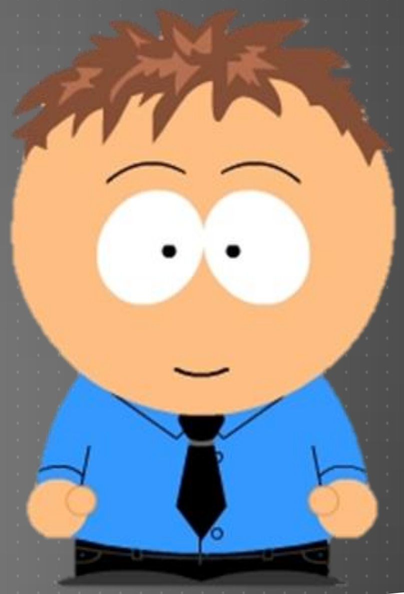
DIFFERENTIATION AND INTEGRATION PART 2

Mr C's IB Standard Notes

In this PDF you can find the following:

Make sure you read through everything and the try examples for yourself before looking at the solutions



THE CHAIN RULE

Differentiating Composite Functions

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Differentiating Composite Functions

Whilst it's easy to differentiate functions like x^5, $(x+2)^5$ would take a very long time (if multiplying out brackets first)

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Differentiating Composite Functions

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<u>Differentiating Composite Functions</u>

$E.g.\ (2x+1)^5, \quad \sin(3x)\,, \quad e^{x^3}$

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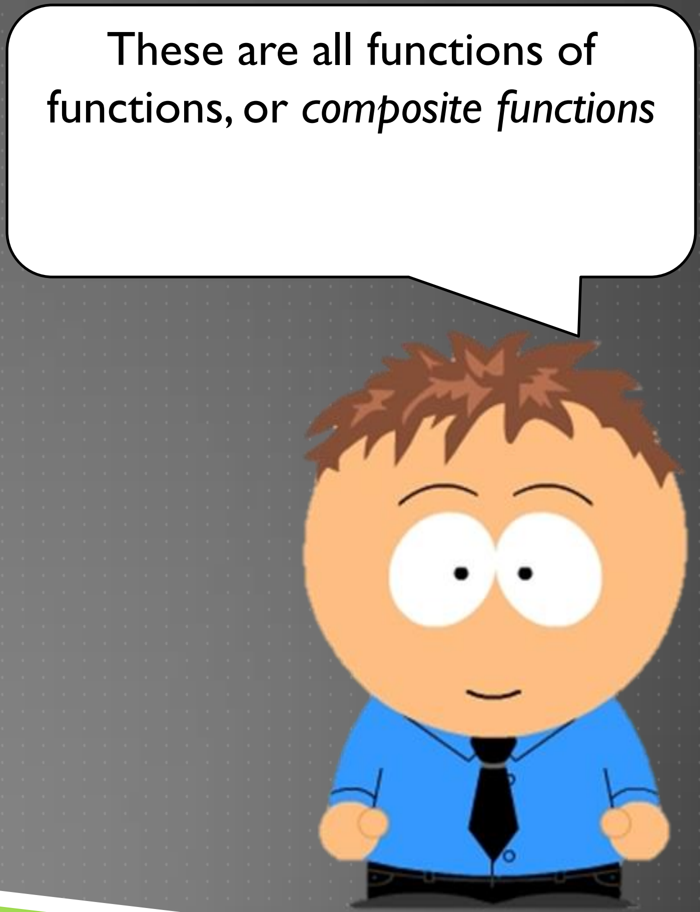

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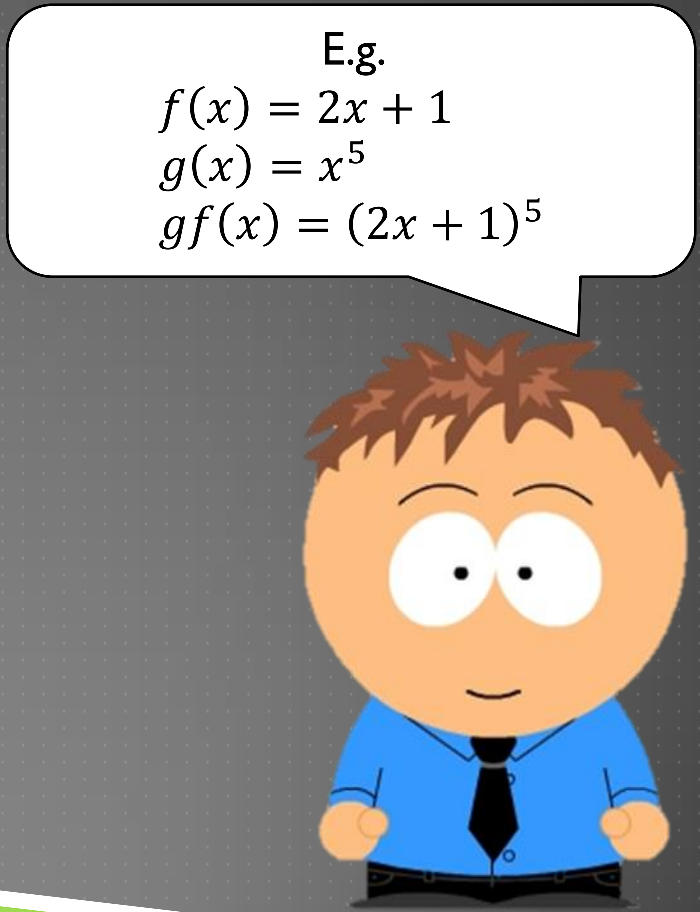

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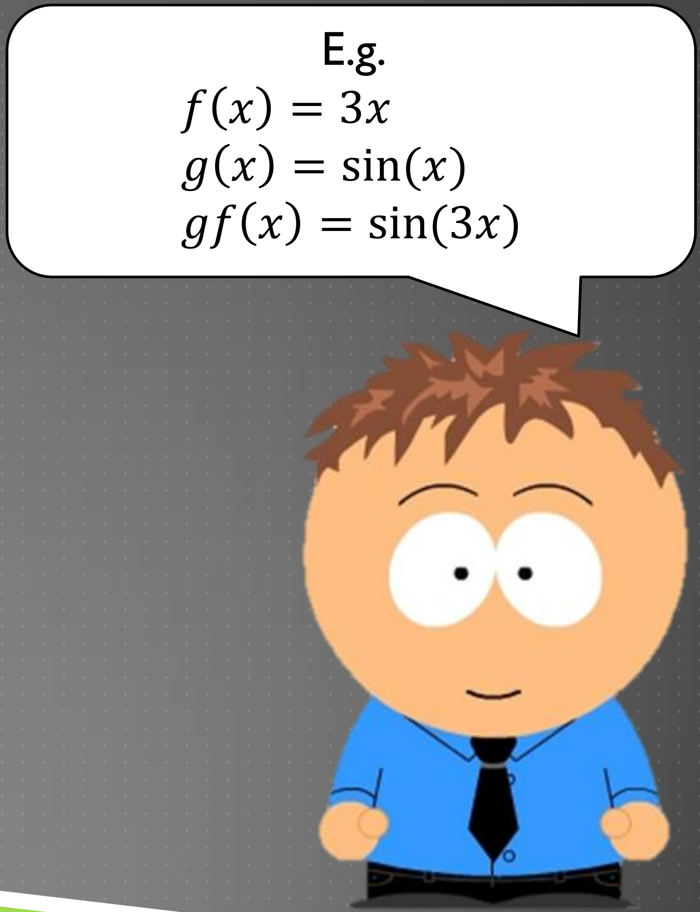

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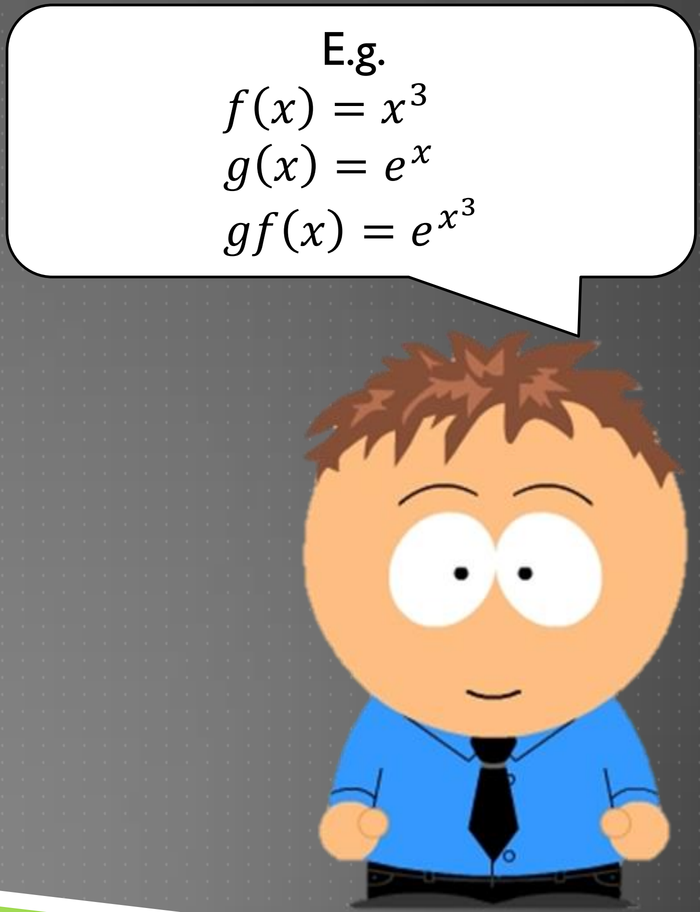

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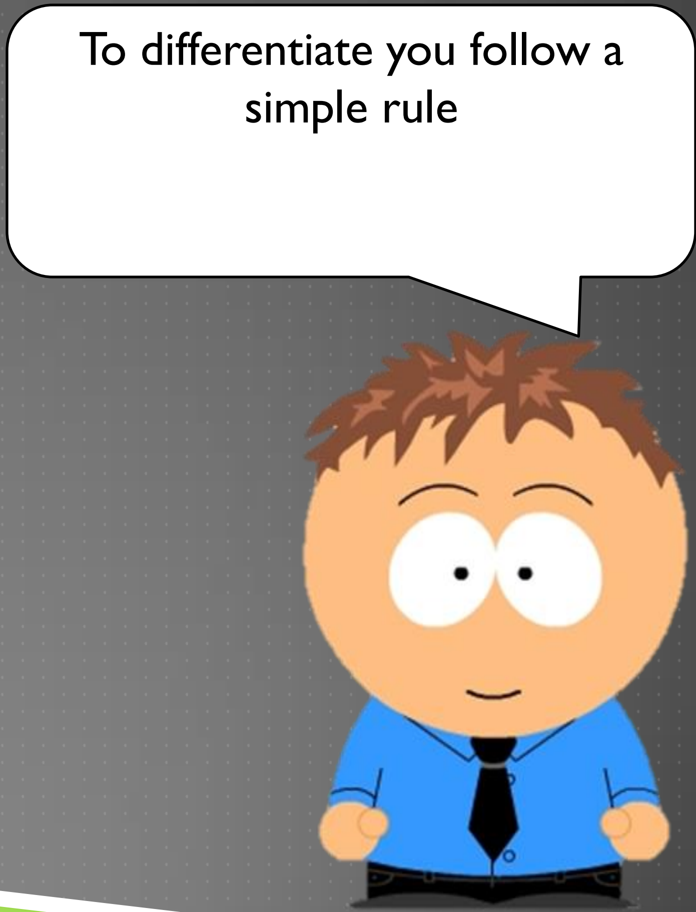
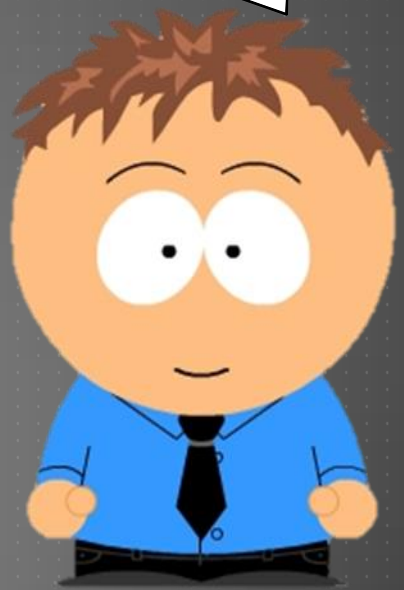


THE CHAIN RULE

Differentiating Composite Functions

$E.g.\ (2x+1)^5, \quad \sin(3x), \quad e^{x^3}$

1. Differentiate the *inner* function, f(x)



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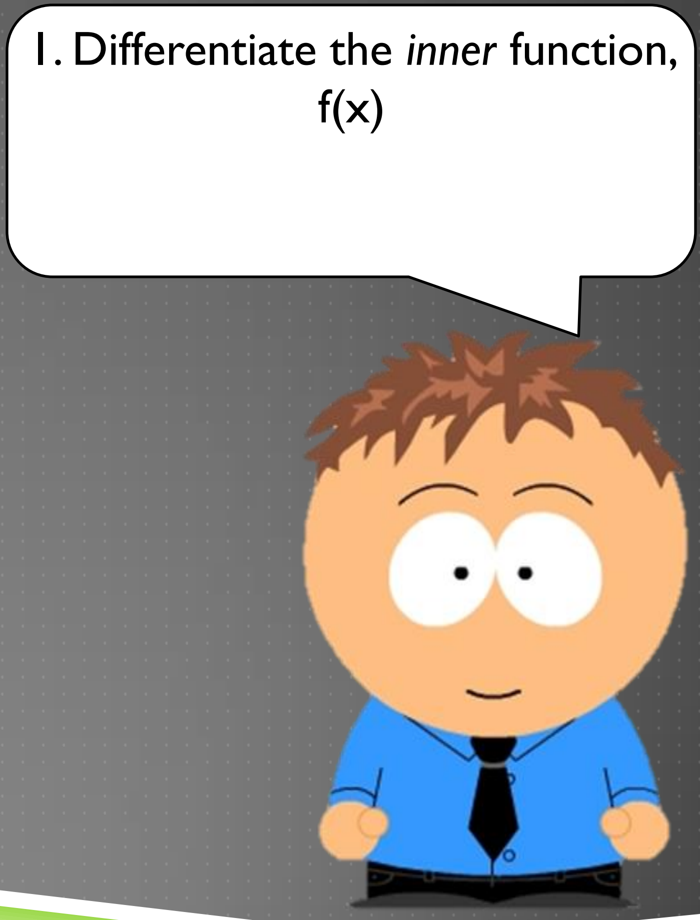
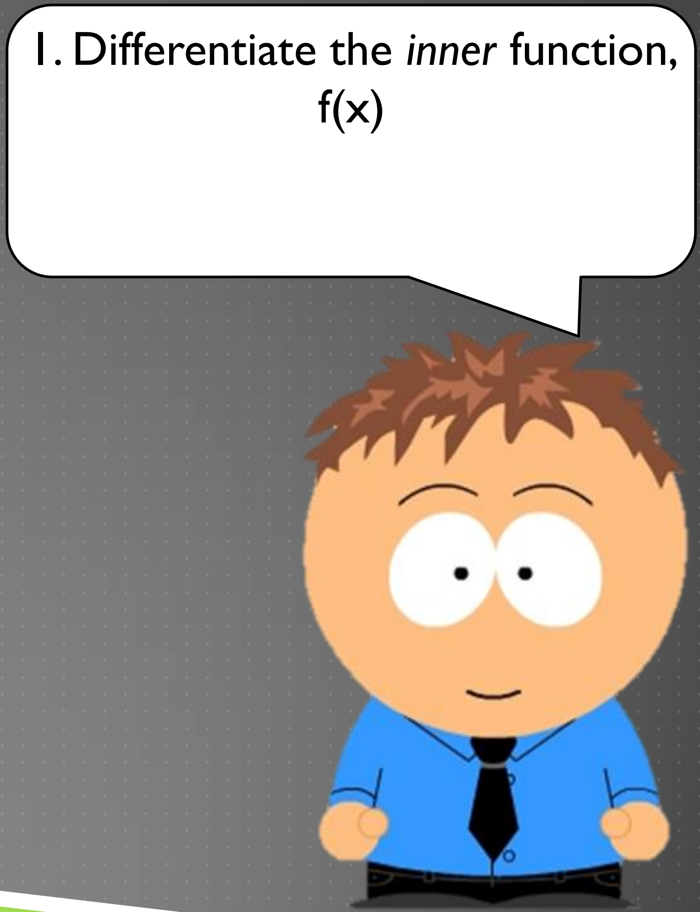


THE CHAIN RULE

Differentiating Composite Functions

$E.g.\ (2x+1)^5, \qquad \sin(3x), \qquad e^{x^3}$

$\frac{dy}{dx} = 2 \qquad \frac{dy}{dx} = 3 \qquad \frac{dy}{dx} = 3x^2$



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$E.g.\ (\underbrace{2x+1}_{u})^5,\qquad \sin(\underbrace{3x}_{u}),\qquad e^{\overbrace{x^3}^{u}}$

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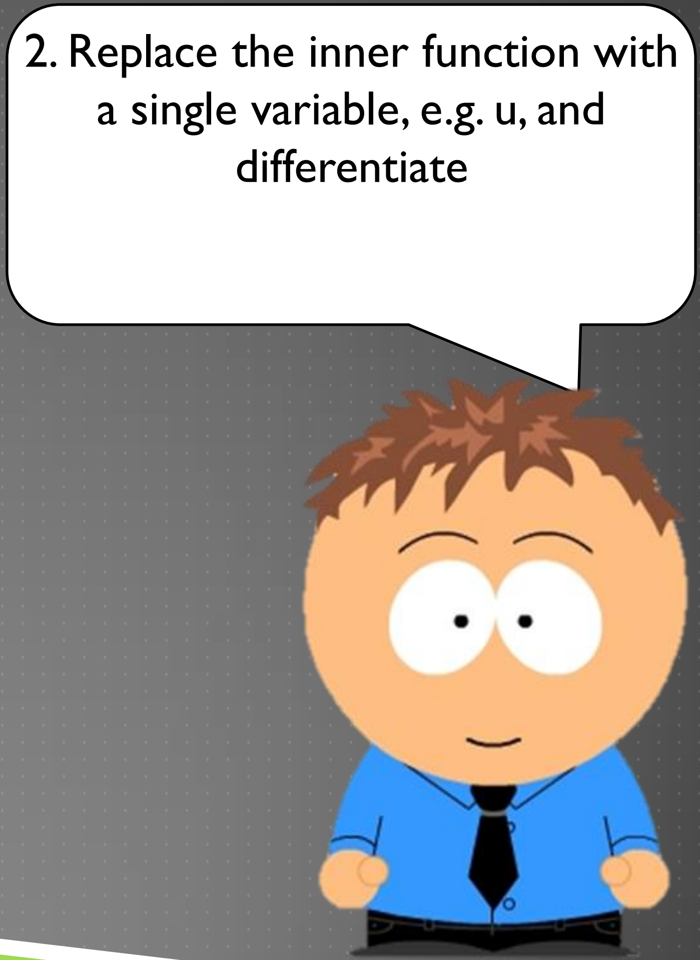


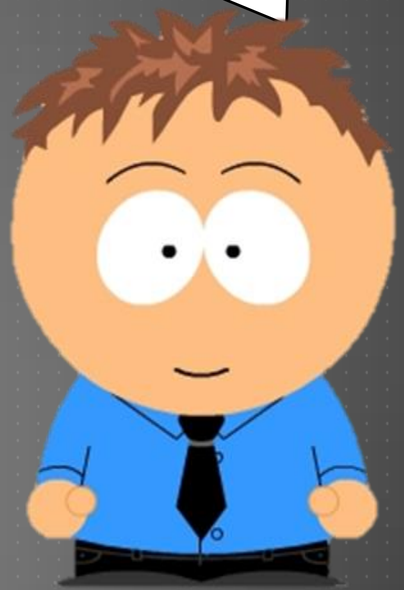
THE CHAIN RULE

<u>Differentiating Composite Functions</u>

$$E.g.\ (\underbrace{2x+1}_{u})^5, \qquad \sin(\underbrace{3x}_{u}), \qquad e^{\overbrace{x^3}^{u}}$$

$$\frac{dy}{dx} = 2\quad 5u^4 \qquad \frac{dy}{dx} = 3\cos(u) \qquad \frac{dy}{dx} = 3x^2\quad e^u$$

2. Replace the inner function with a single variable, e.g. u, and differentiate



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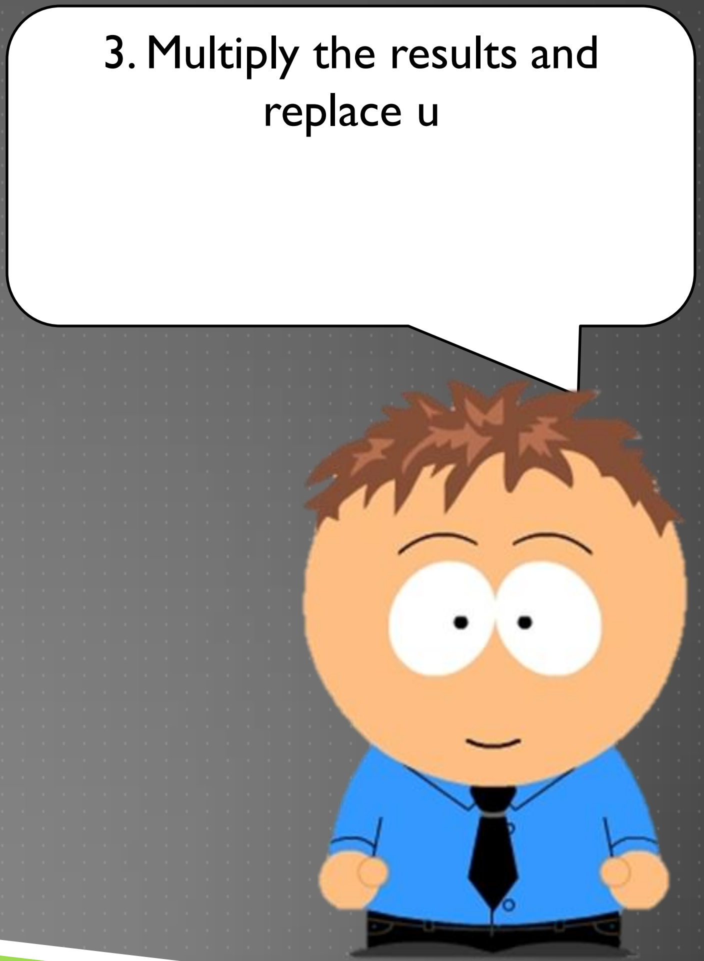

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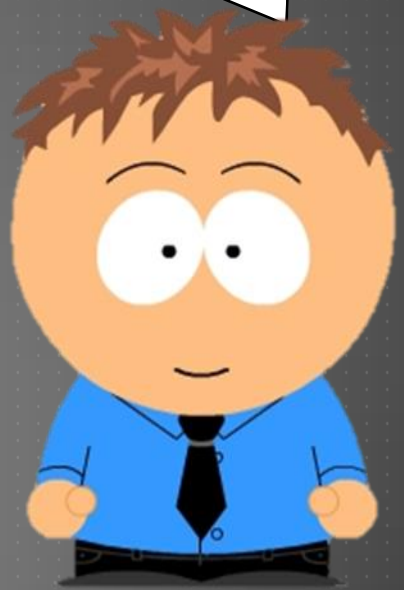
$$E.g.\ (\underbrace{2x+1}_{u})^5, \qquad \sin(\underbrace{3x}_{u})\,, \qquad e^{\overbrace{x^3}^{u}}$$

$$\frac{dy}{dx} = 2 \quad 5u^4 \qquad \frac{dy}{dx} = 3\cos(u) \qquad \frac{dy}{dx} = 3x^2 \quad e^u$$

$$= 10(2x+1)^4 \qquad = 3\cos(3x) \qquad = 3x^2e^{x^3}$$

3. Multiply the results and replace u



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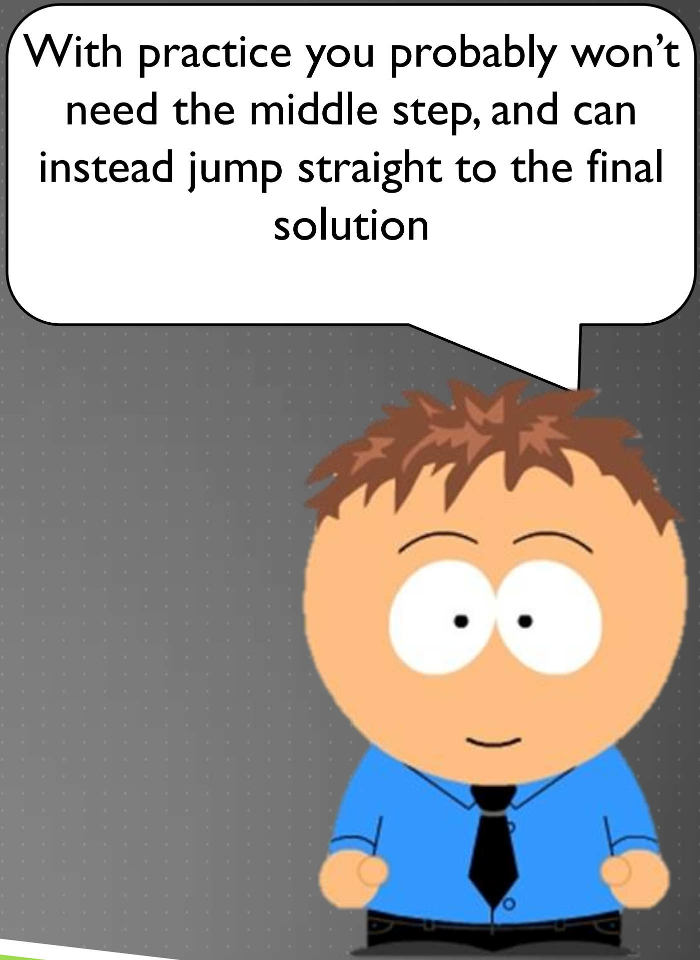
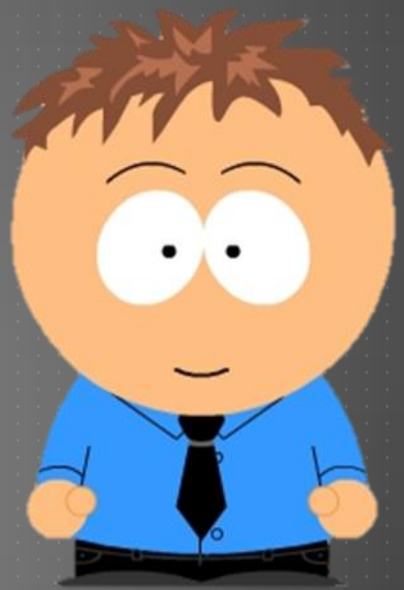


INTEGRATION BY INSPECTION

<u>Reversing the chain rule for integration</u>



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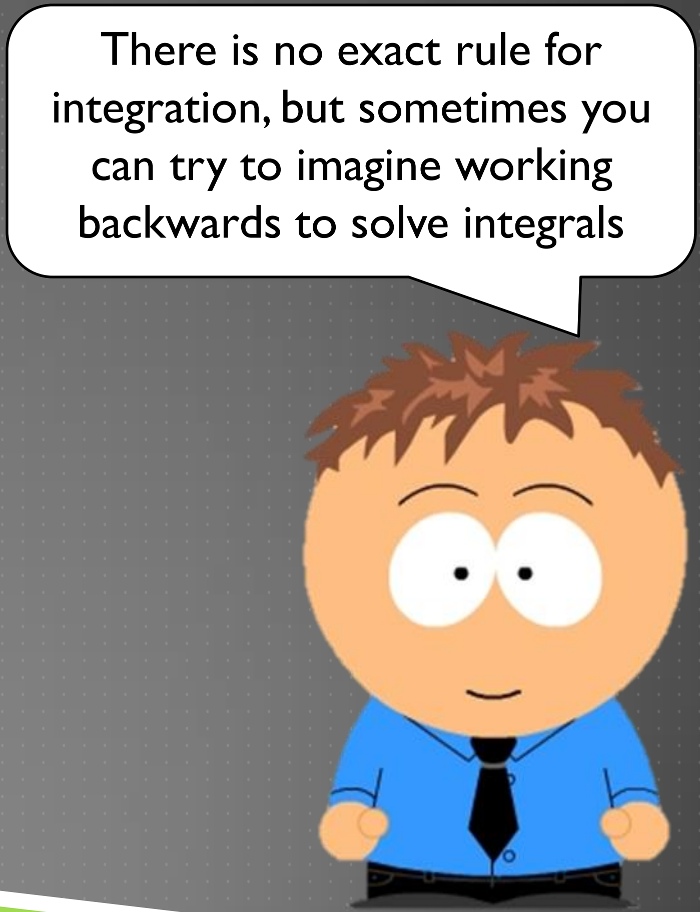
Reversing the chain rule for integration

E.g. Evaluate $\int (2x-3)^4 \quad dx$

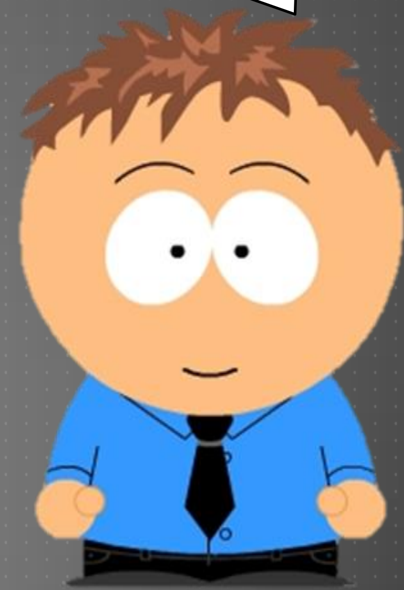


INTEGRATION BY INSPECTION

<u>Reversing the chain rule for integration</u>

E.g. Evaluate $\int (2x-3)^4 \quad dx$

Try to think about what you would have to differentiate to get the result in the question



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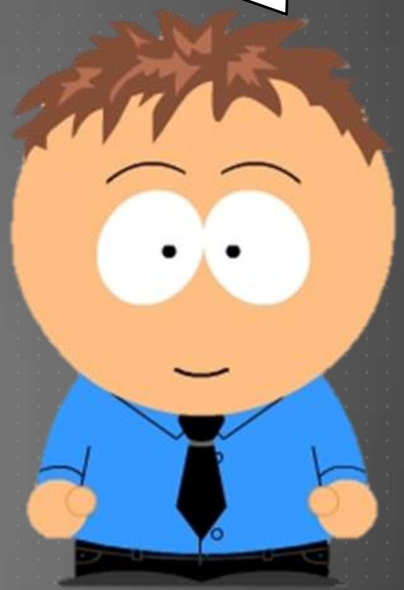


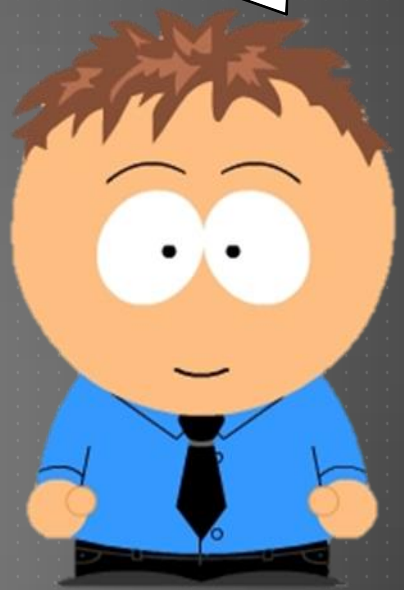
INTEGRATION BY INSPECTION

Reversing the chain rule for integration

E.g. Evaluate $\int (2x-3)^4 \quad dx$

?

We know to end up with
$(2x-3)^{4}$
we need something like
$(2x-3)^{5}$



INTEGRATION BY INSPECTION

<u>Reversing the chain rule for integration</u>

E.g. Evaluate $\int (2x-3)^4 \quad dx$

$$(2x-3)^5$$

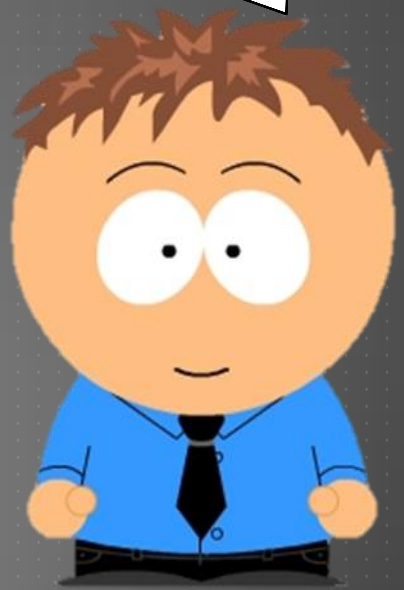
?

We know to end up with

$(2x-3)^{4}$

we need something like

$(2x-3)^{5}$



INTEGRATION BY INSPECTION

<u>Reversing the chain rule for integration</u>

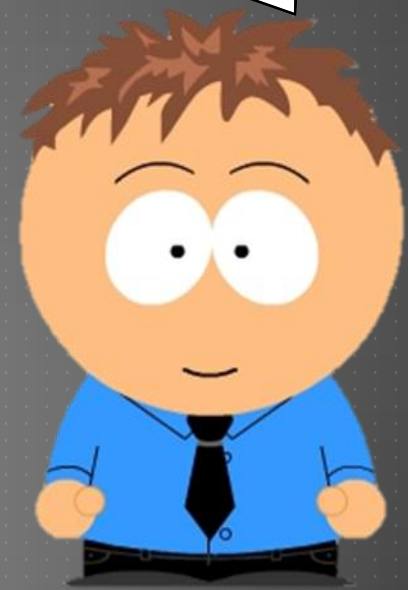
E.g. Evaluate $\int (2x-3)^4 \quad dx$

$$2(2x-3)^4$$

$$\boxed{(2x-3)^5}$$

?

When we differentiate
$(2x-3)^5$
We get
$2(2x-3)^4$



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<u>Reversing the chain rule for integration</u>

E.g. Evaluate $\int (2x-3)^4 \quad dx$

$2(2x-3)^4$

$$\boxed{(2x-3)^5}$$

?

INTEGRATION BY INSPECTION

Reversing the chain rule for integration

E.g. Evaluate $\int (2x-3)^4 \quad dx$

$$\cancel{\frac{1}{2}}\cancel{2}(2x-3)^4$$

$$\boxed{\frac{1}{2}(2x-3)^5}$$

?

INTEGRATION BY INSPECTION

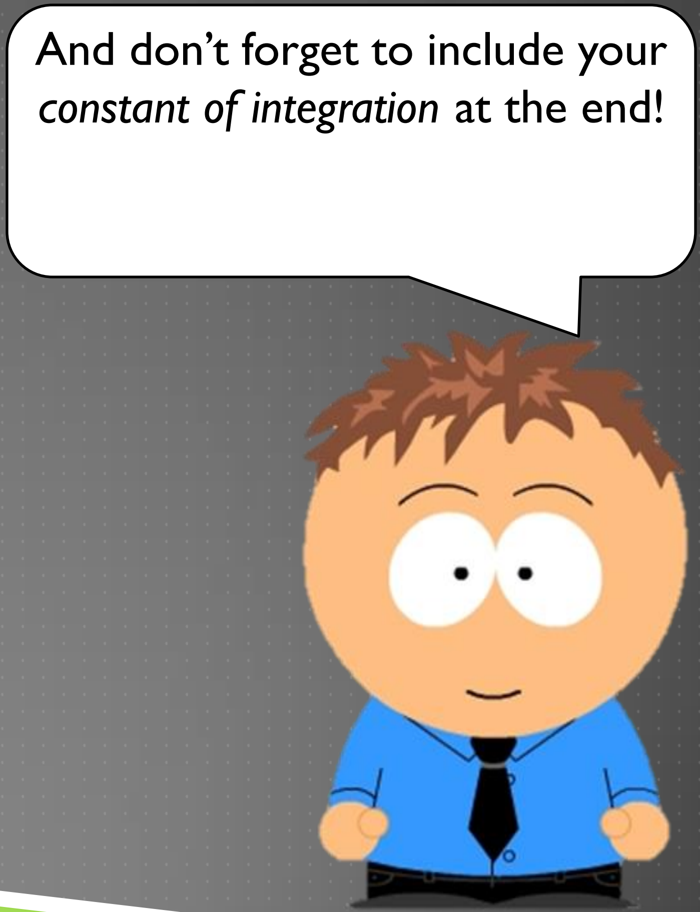
<u>Reversing the chain rule for integration</u>

E.g. Evaluate $\int (2x-3)^4 \quad dx$

$$\cancel{\frac{1}{2}}\cancel{2}(2x-3)^4$$

$$\boxed{\frac{1}{2}(2x-3)^5 + \boldsymbol{C}}$$

?



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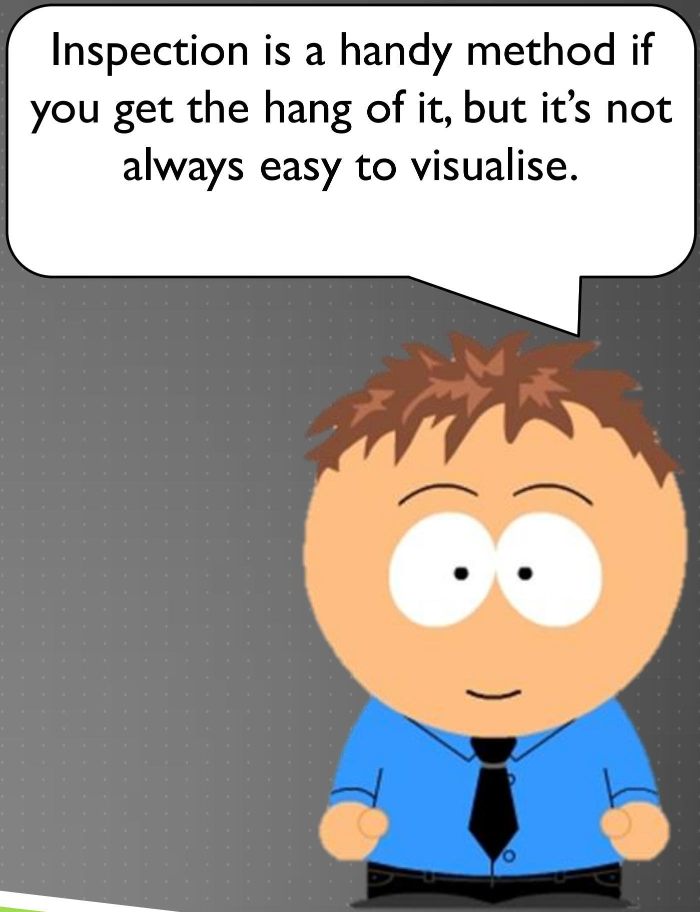

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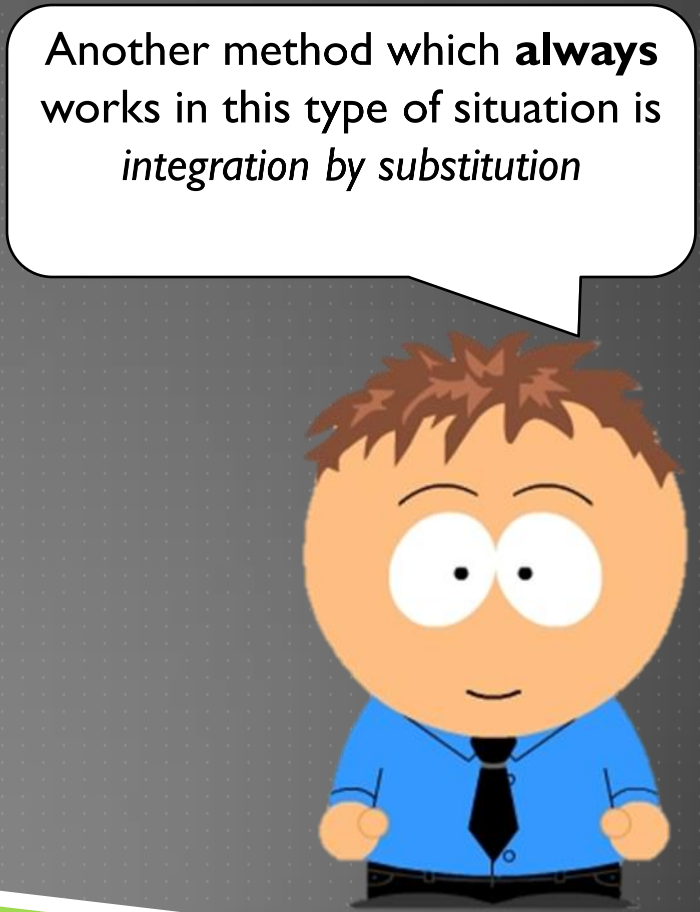

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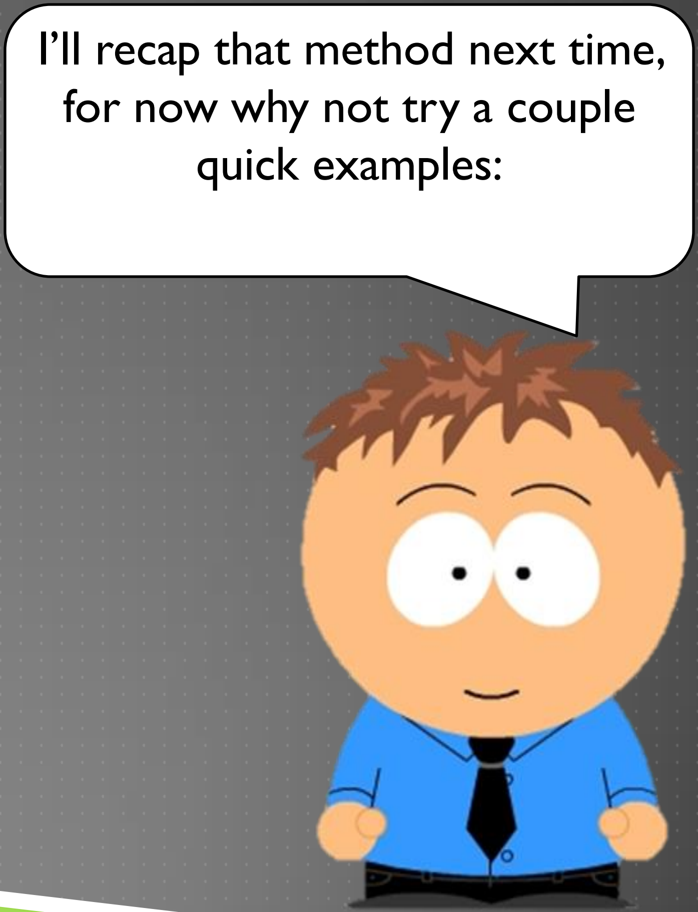


EXAMPLES

What values of x does the function $f(x) = \ln(x^2 - 35)$ have a gradient of 1?

1.

Find the area under the curve $y = \frac{1}{2}e^{4x}$, bounded by the x-axis, and the lines x=0 and x=1

2.

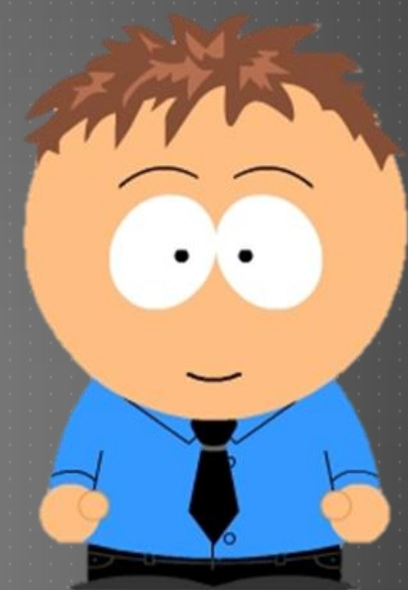
SOLUTIONS

What values of x does the function $f(x) = \ln(x^2 - 35)$ have a gradient of 1?

1.

$f(x) = \ln(x^2 - 35)$

We need to find gradient, which means we must differentiate



SOLUTIONS

What values of x does the function $f(x) = \ln(x^2 - 35)$ have a gradient of 1?

1.

$f(x) = \ln(x^2 - 35)$

This is a composite function, so use the chain rule

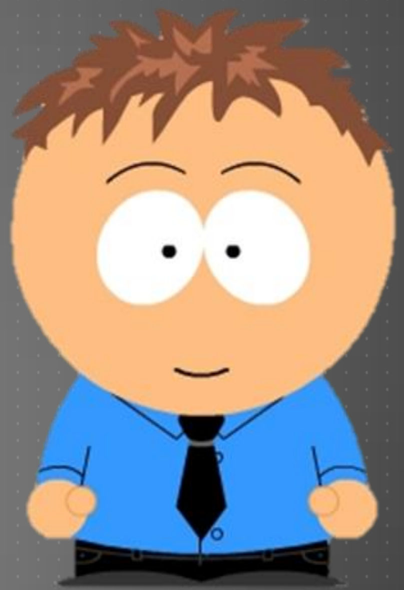
SOLUTIONS

What values of x does the function $f(x) = \ln(x^2 - 35)$ have a gradient of 1?

1.

$f(x) = \ln(x^2 - 35)$

Differentiate the inner function



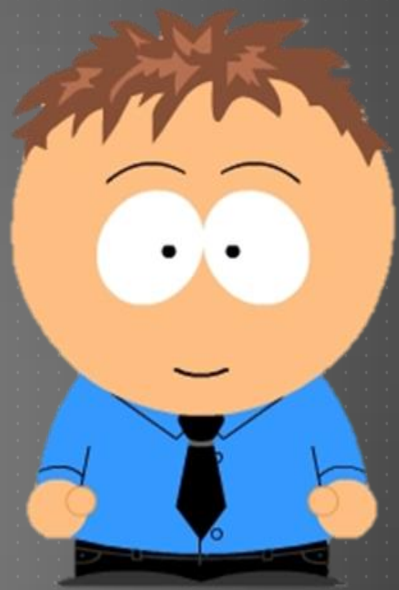
SOLUTIONS

What values of x does the function $f(x) = \ln(x^2 - 35)$ have a gradient of 1?

1.

$$f(x) = \ln(x^2 - 35)$$

$$f'(x) = 2x$$



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What values of x does the function $f(x) = \ln(x^2 - 35)$ have a gradient of 1?

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$$f(x) = \ln\underbrace{(x^2 - 35)}_{u}$$

$$f'(x) = 2x$$

Differentiate the outer

SOLUTIONS

What values of x does the function $f(x) = \ln(x^2 - 35)$ have a gradient of 1?

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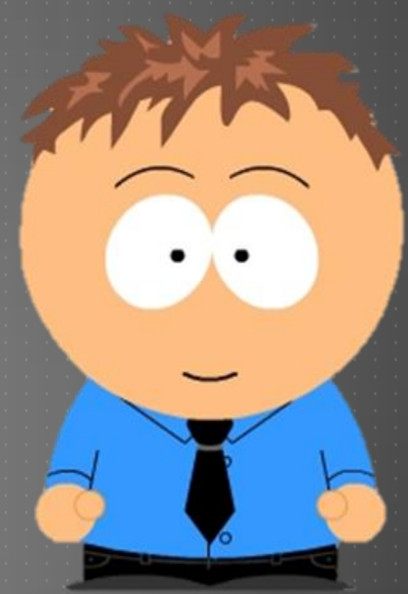
$$f(x) = \ln\underbrace{(x^2 - 35)}_{u}$$

$$f'(x) = 2x \, \frac{1}{x^2 - 35}$$

Remember the derivative of $\ln(x)$ is $\frac{1}{x}$



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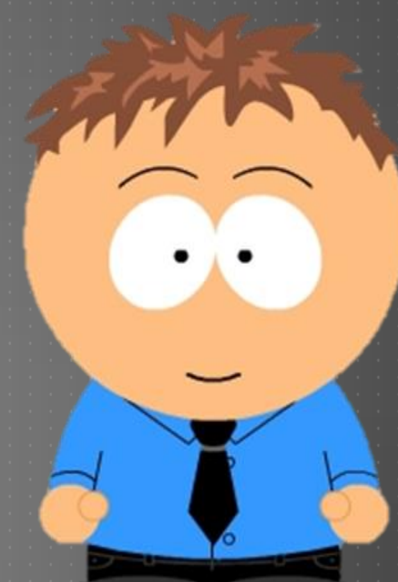

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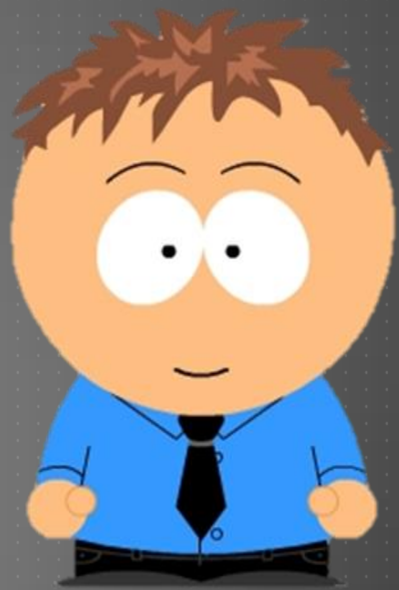
$f'(x) = 2x \dfrac{1}{x^2 - 35}$

$f'(x) = \dfrac{2x}{x^2-35}$

$\dfrac{2x}{x^2-35} = 1$

We want to know when the gradient (derivative) is 1



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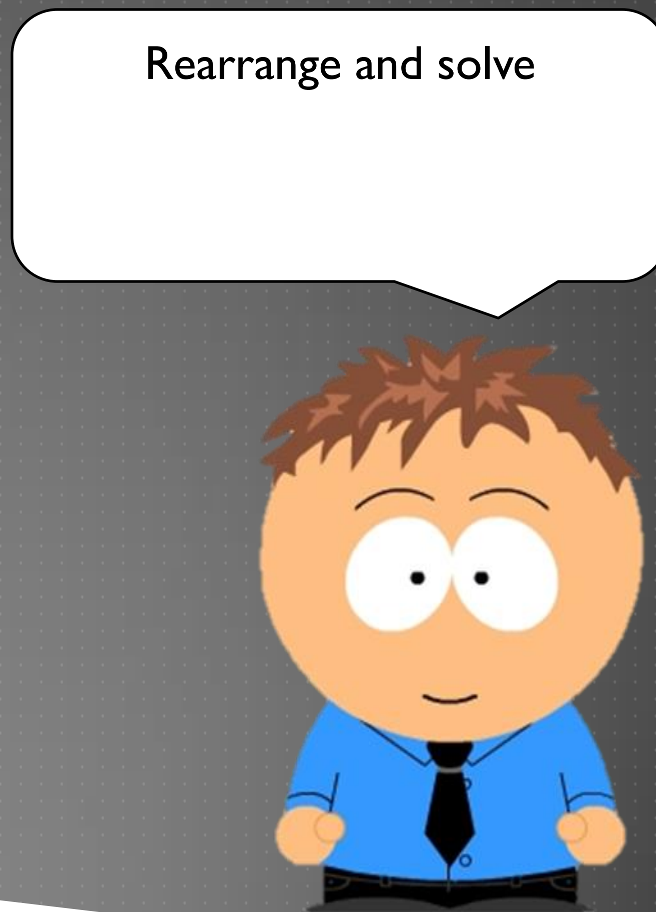

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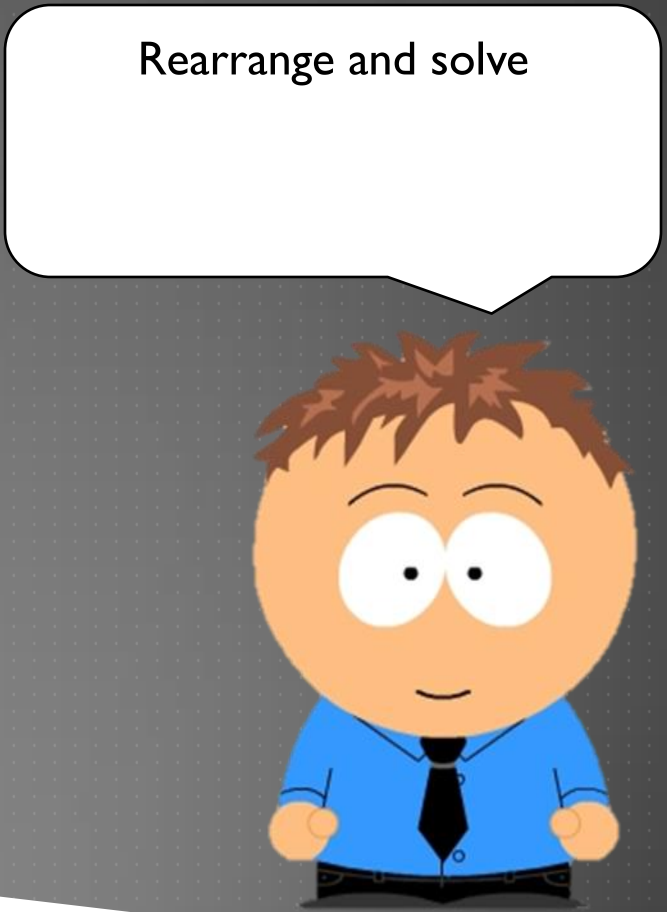

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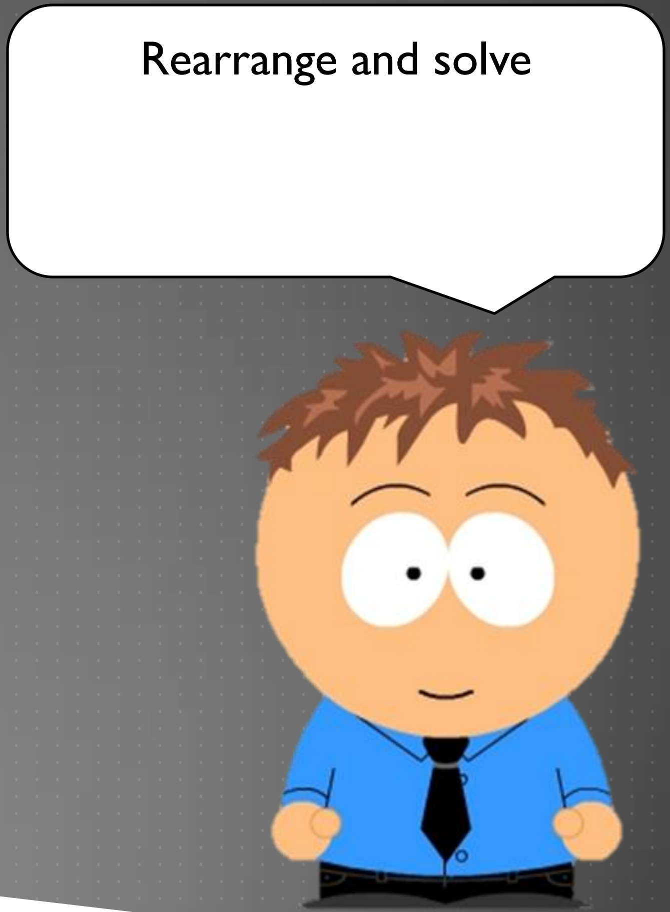

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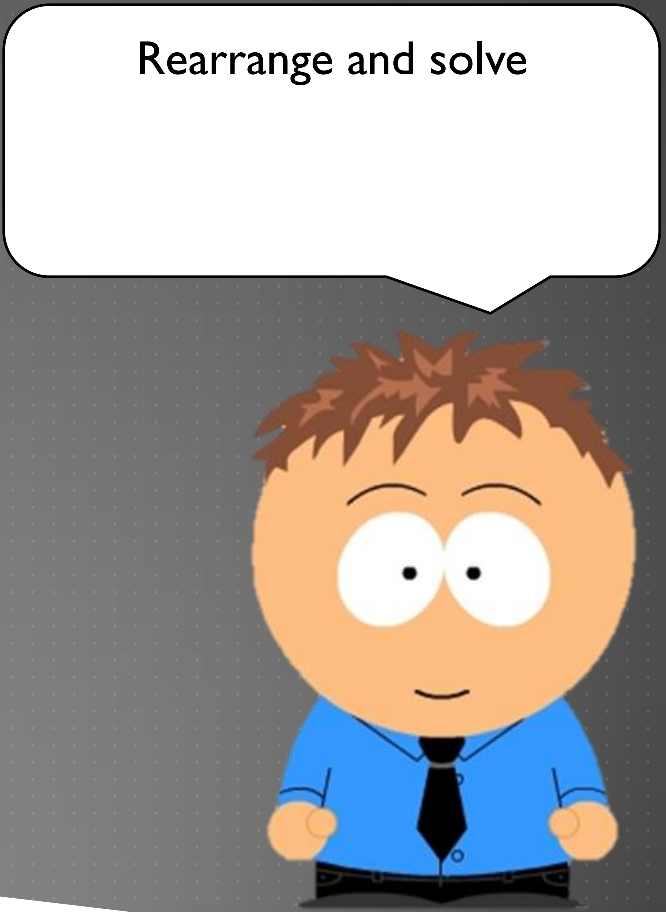
$$x = 7 \text{ or } x = -5$$



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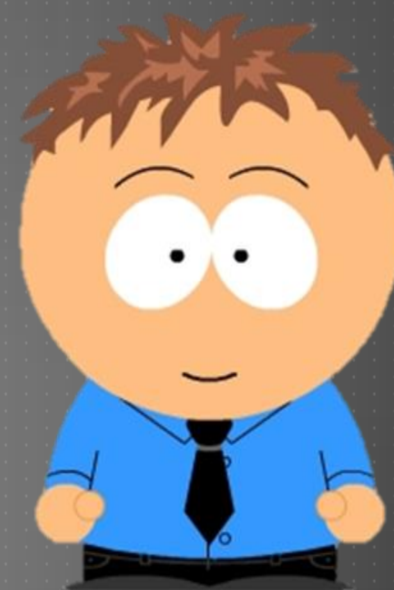
We have 2 solutions, but we must remember to be careful when dealing with ln graphs



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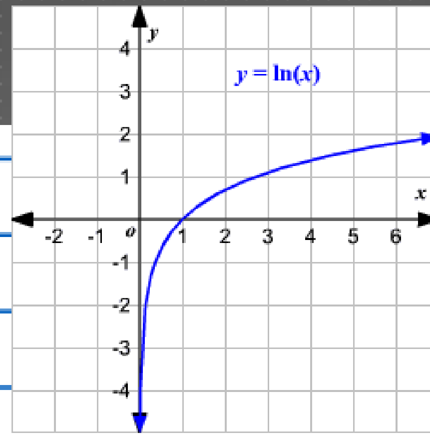
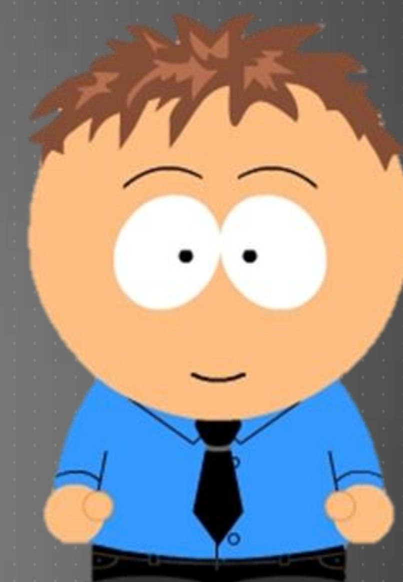


$$\underbrace{ln(x^2 - 35)}_{}$$

Only exists if this part is > 0



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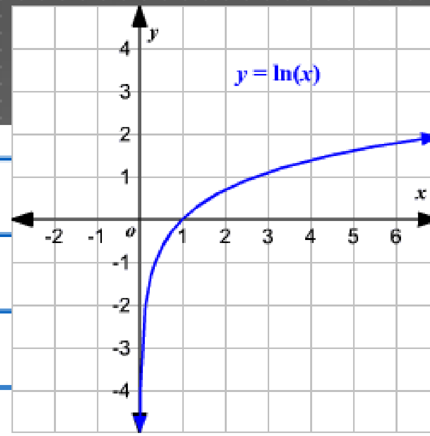


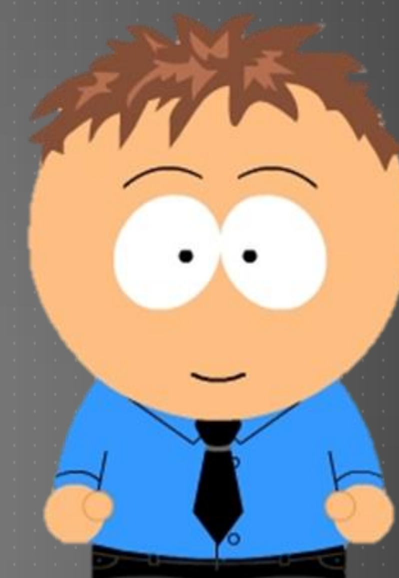
$$f'(x) = \frac{2x}{x^2 - 35} \qquad \frac{2x}{x^2 - 35} = 1$$

$$2x = x^2 - 35$$

$$0 = x^2 - 2x - 35$$

$$\boldsymbol{x = 7}\ \cancel{or\ x = -5}$$

Therefore x=7 is the only solution

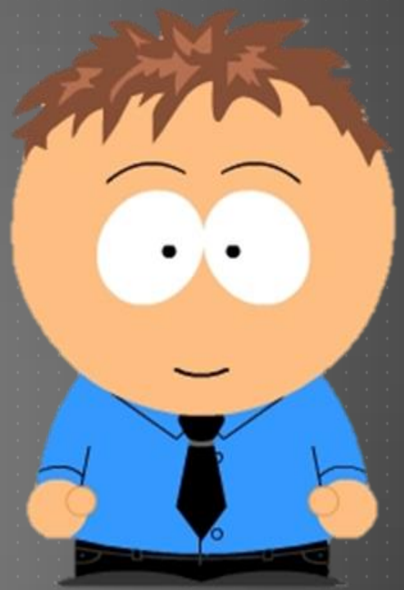


SOLUTIONS

Find the area under the curve $y = \frac{1}{2}e^{4x}$, bounded by the x-axis, and the lines y=0 and y=1

2.

We need to find an area, so that means we must integrate our function



SOLUTIONS

Find the area under the curve $y = \frac{1}{2}e^{4x}$, bounded by the x-axis, and the lines y=0 and y=1

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$$\int \frac{1}{2}e^{4x}dx$$

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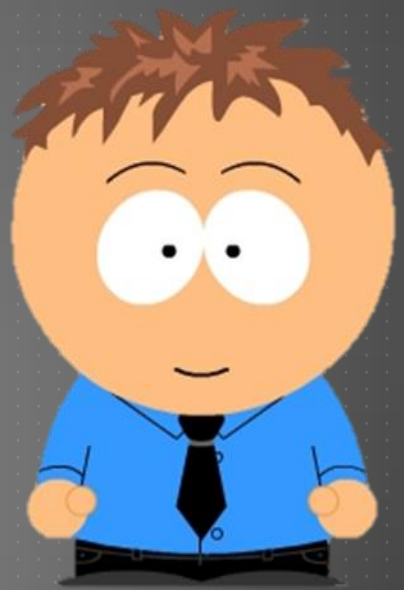

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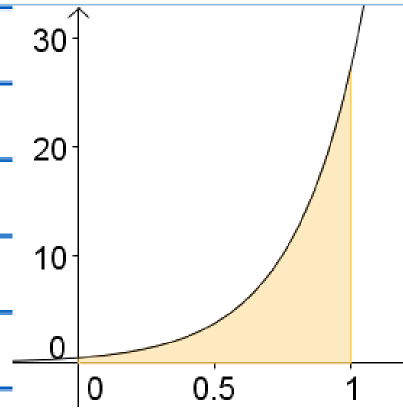
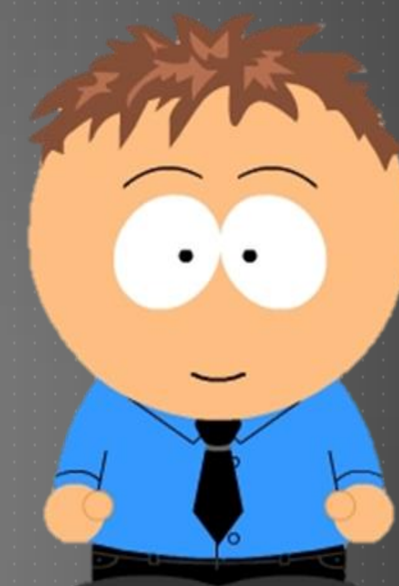


A quick sketch shows the area we are looking for. There is no section under the x-axis so we can just use limits 0 and 1



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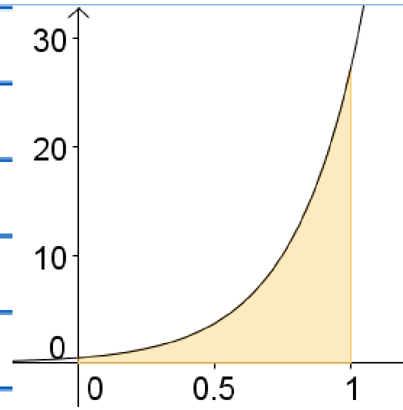
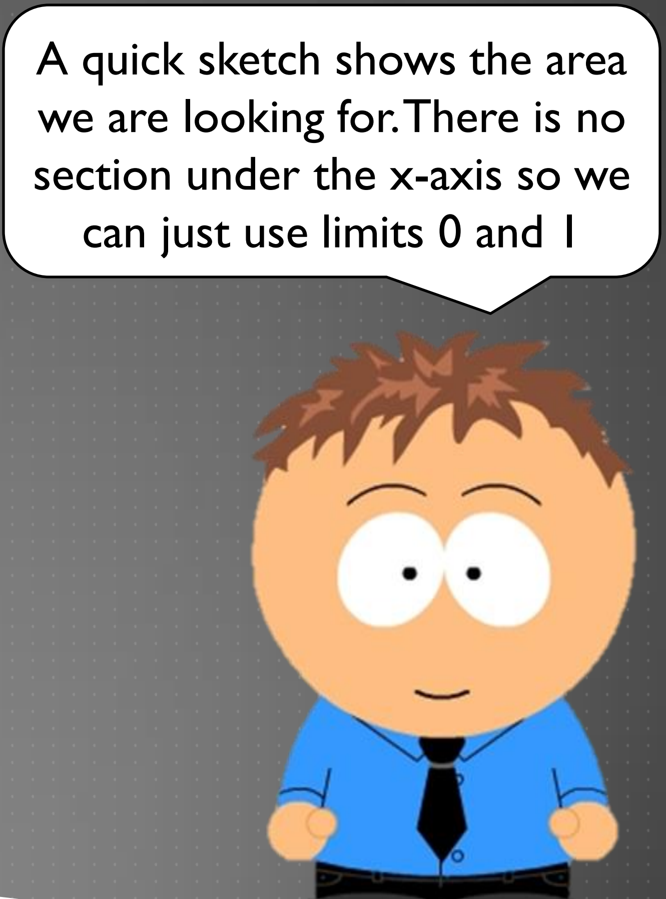
$$\int_{0}^{1} \frac{1}{2}e^{4x}\,dx$$





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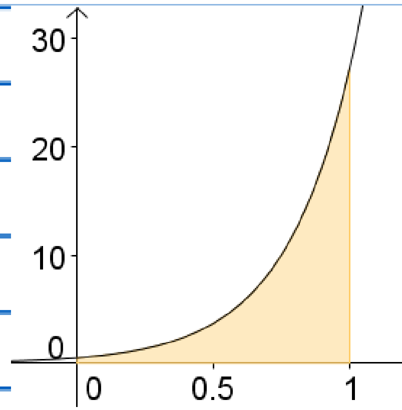

By thinking about what would differentiate to give our answer, quite quickly we can figure out the integral

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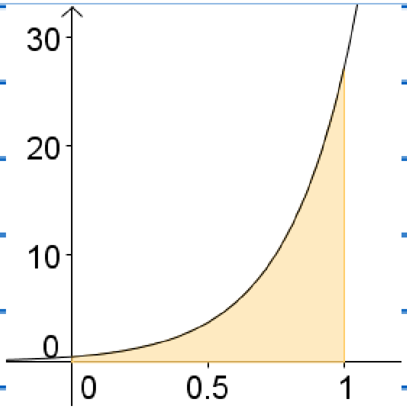
$$\int_0^1 \frac{1}{2}e^{4x}\,dx$$

$$= \left[\frac{1}{8}e^{4x}\right]_0^1$$



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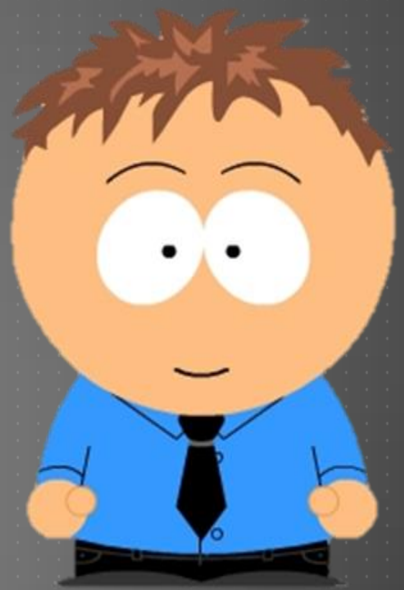

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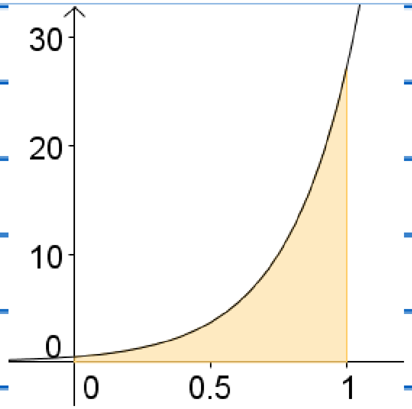
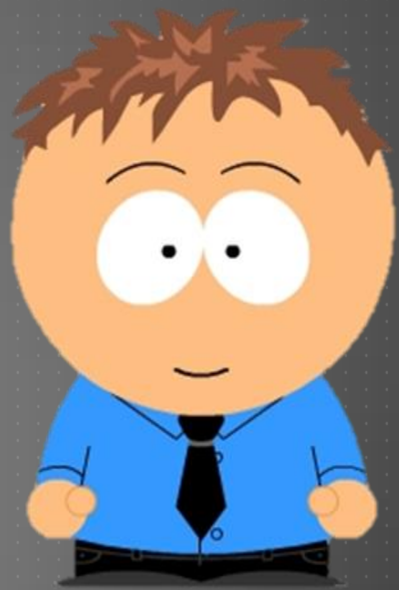


And all that's left to do is evaluate the limits



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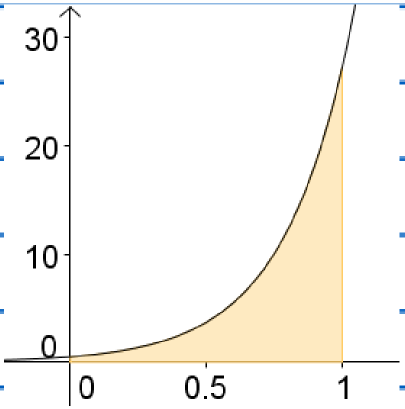

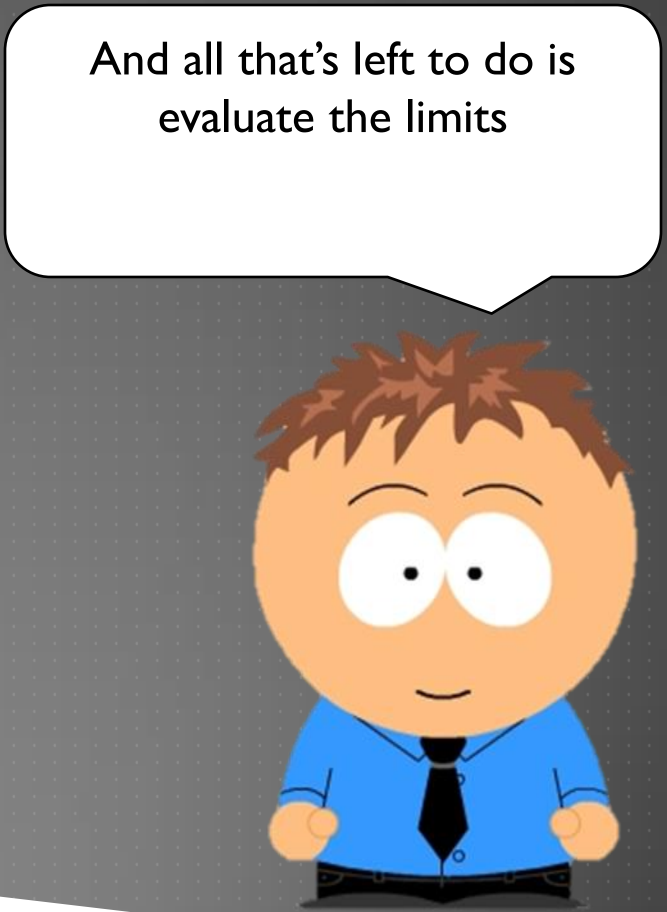

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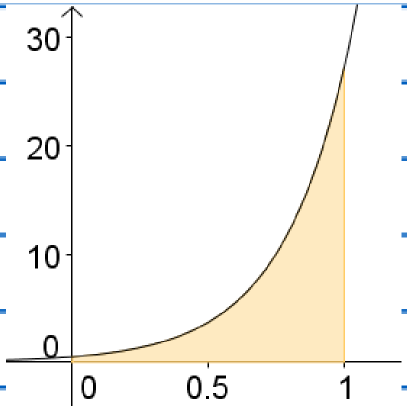

Remembering that $e^0 = 1$…

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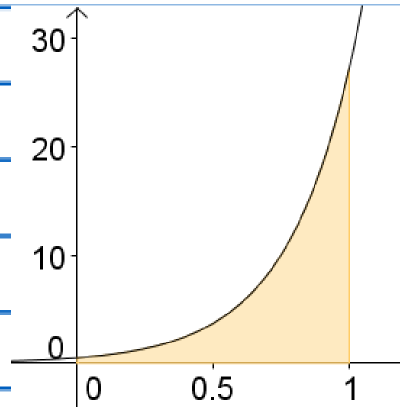

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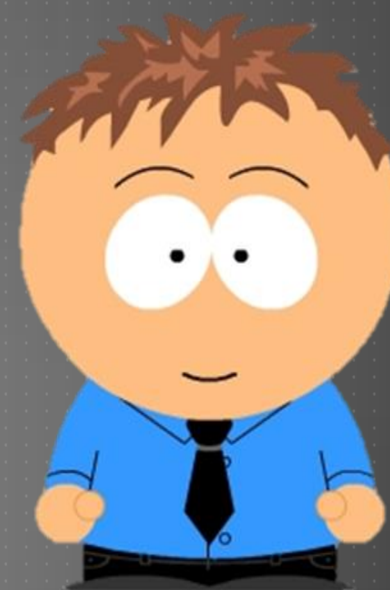

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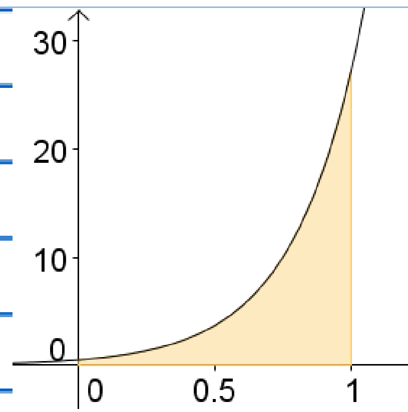

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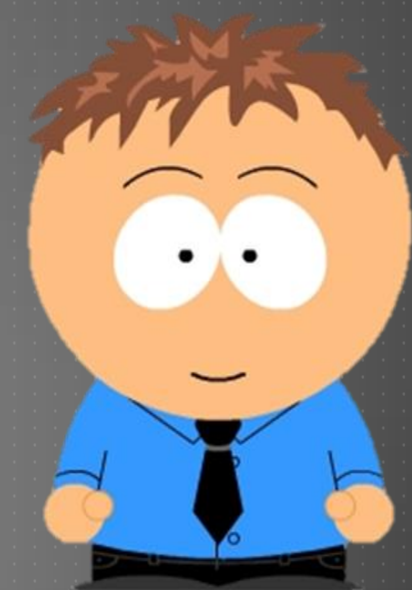
You can leave your answer in this form, or evaluate on the GDC



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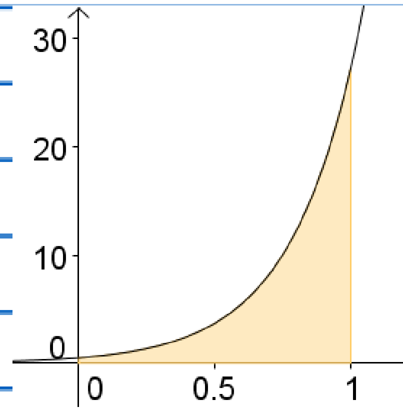
$$= \frac{1}{8}e^{4} \quad -1 \quad = 5.82 \text{ (3sf)}$$



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