

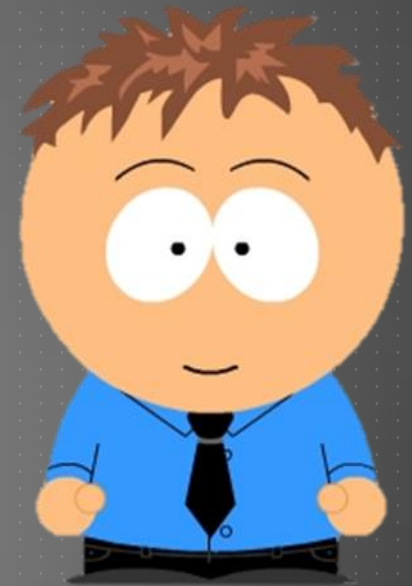
DIFFERENTIATION AND INTEGRATION PART 3

Mr C's IB Standard Notes

In this PDF you can find the following:

1. [The Product Rule](#)
2. [The Quotient Rule](#)
3. [Integration by Substitution](#)
4. [Worked Examples](#)

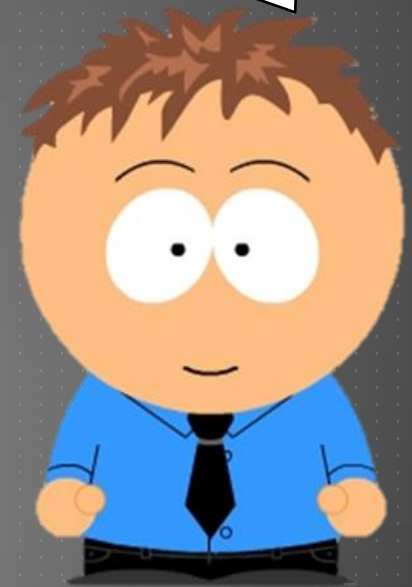
Make sure you read through everything and then try examples for yourself before looking at the solutions



THE PRODUCT RULE

Differentiating when one function is multiplied by another

Now we've seen functions like x^5 and $(x + 2)^5$, we move on to two further examples. Each has their own rule for differentiation

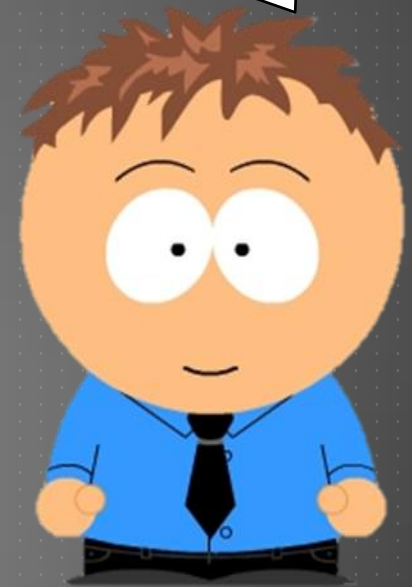


THE PRODUCT RULE

Differentiating when one function is multiplied by another

We start with functions that have been multiplied together, e.g.

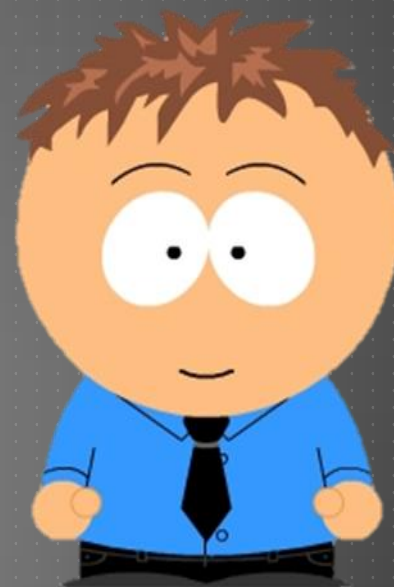
$$f(x) = e^{3x} \sin(x)$$



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

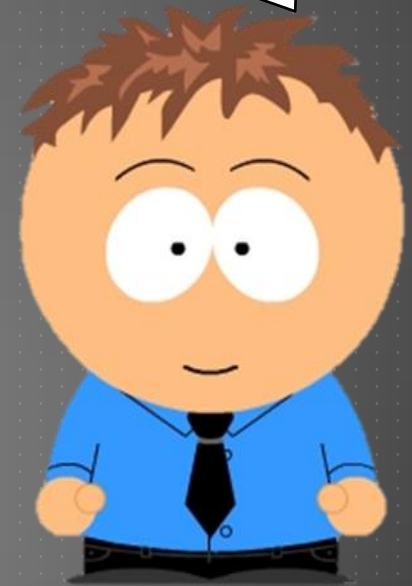


THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

We have two functions which we can label, for example, u and v



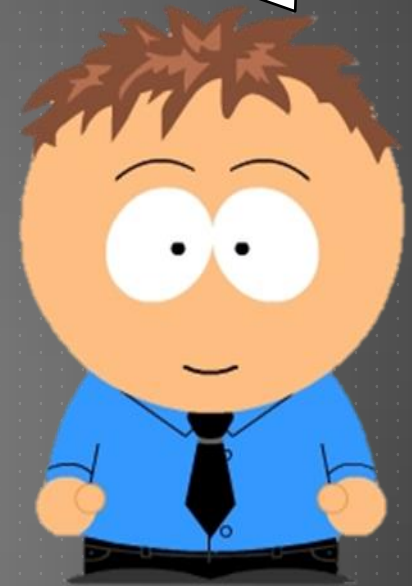
THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$

We have two functions which we can label, for example, u and v



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$

$$v = \sin(x)$$

We have two functions which we can label, for example, u and v



THE PRODUCT RULE

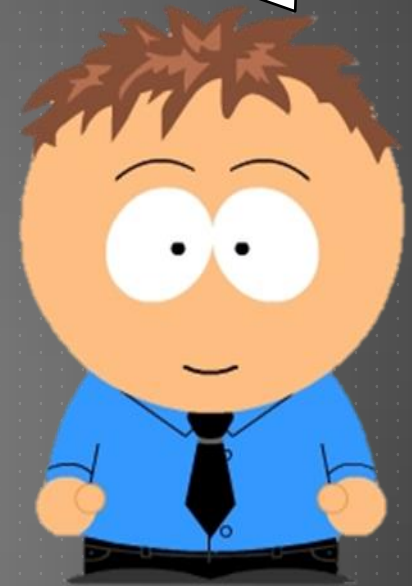
Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$

$$v = \sin(x)$$

The formula book tells us how to differentiate the product of $u \times v$ using method is called the *product rule*



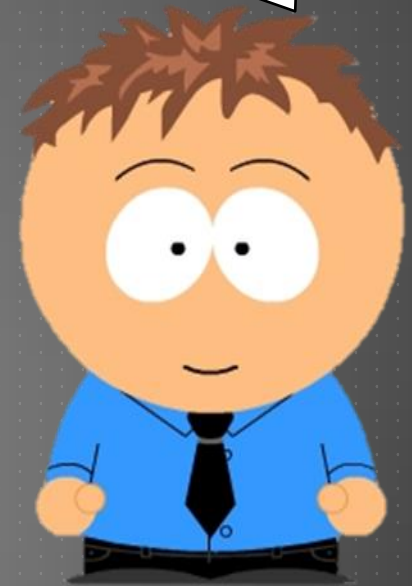
THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x} \quad v = \sin(x)$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



THE PRODUCT RULE

Differentiating when one function is multiplied by another

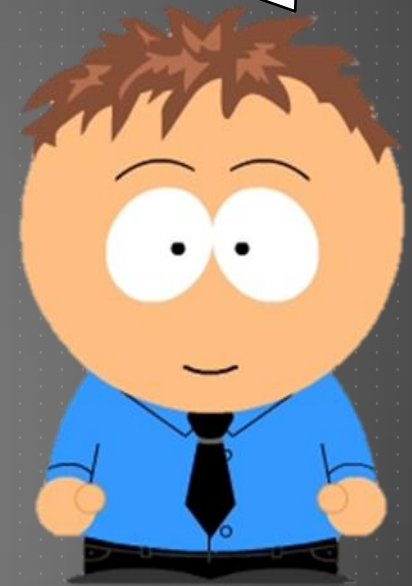
$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$

$$v = \sin(x)$$

First differentiate both u and v
with respect to x

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$

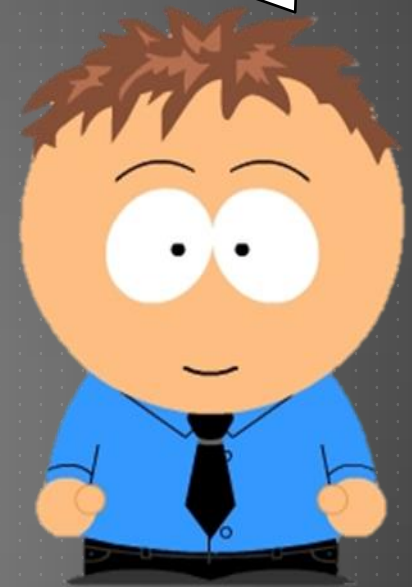


$$v = \sin(x)$$



First differentiate both u and v
with respect to x

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$



$$\frac{du}{dx} = 3e^{3x}$$

$$v = \sin(x)$$



$$\frac{dv}{dx} = \cos(x)$$

First differentiate both u and v
with respect to x

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$



$$\frac{du}{dx} = 3e^{3x}$$

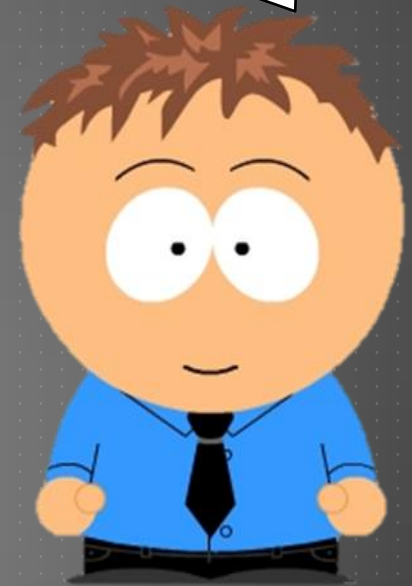
$$v = \sin(x)$$



$$\frac{dv}{dx} = \cos(x)$$

Then substitute the expressions
into the equation

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$



$$\frac{du}{dx} = 3e^{3x}$$

$$v = \sin(x)$$

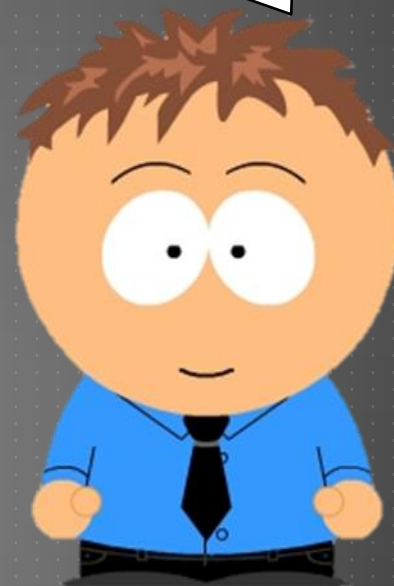


$$\frac{dv}{dx} = \cos(x)$$

Then substitute the expressions
into the equation

$$f'(x) = e^{3x} \times \cos(x) + \sin(x) \times 3e^{3x}$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$



$$\frac{du}{dx} = 3e^{3x}$$

$$v = \sin(x)$$

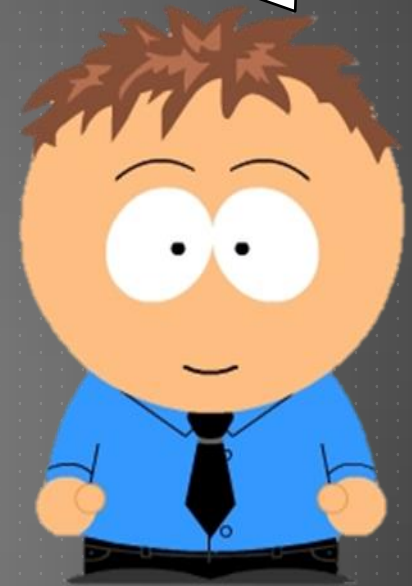


$$\frac{dv}{dx} = \cos(x)$$

$$f'(x) = e^{3x} \times \cos(x) + \sin(x) \times 3e^{3x}$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

The final step is to simplify and tidy up where possible



THE PRODUCT RULE

Differentiating when one function is multiplied by another

$$f(x) = e^{3x} \sin(x)$$

$$u = e^{3x}$$



$$\frac{du}{dx} = 3e^{3x}$$

$$v = \sin(x)$$



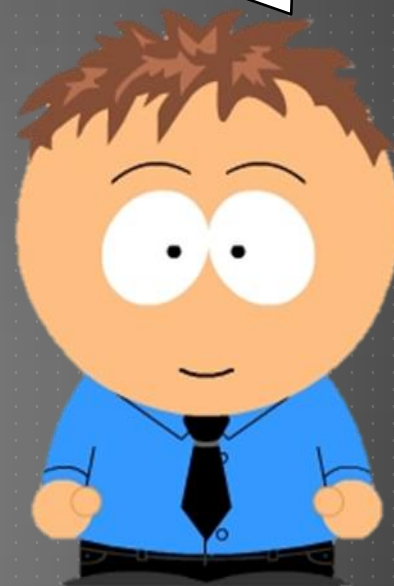
$$\frac{dv}{dx} = \cos(x)$$

$$f'(x) = e^{3x} \times \cos(x) + \sin(x) \times 3e^{3x}$$

$$f'(x) = e^{3x} (\cos(x) + 3 \sin(x))$$

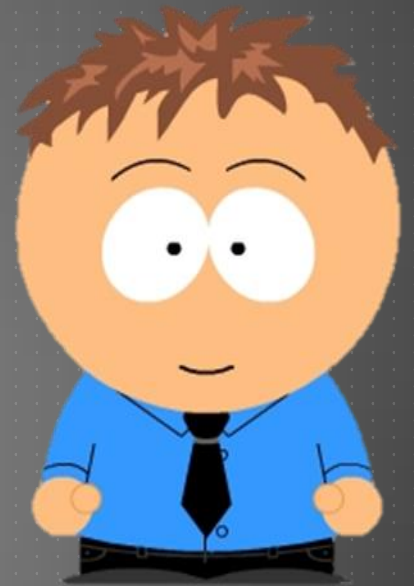
$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

The final step is to simplify and tidy up where possible



THE QUOTIENT RULE

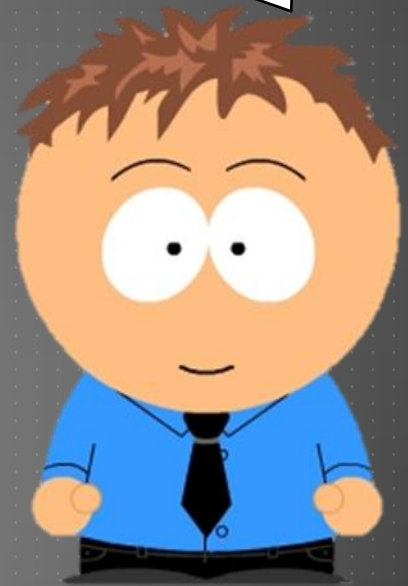
Differentiating when one function is divided by another



THE QUOTIENT RULE

Differentiating when one function is divided by another

Similar to the previous example,
there is another formula that
deals when one function is
divided by another

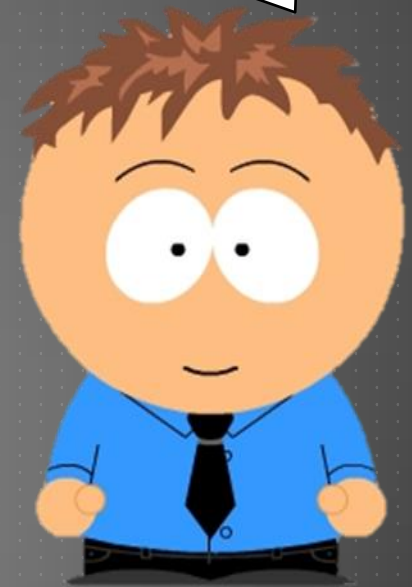


THE QUOTIENT RULE

Differentiating when one function is divided by another

For example, take the function

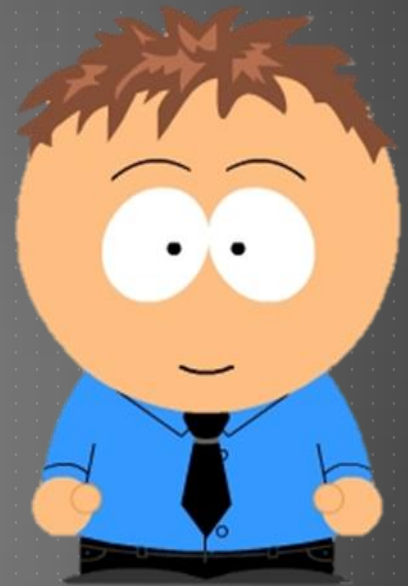
$$\frac{x^2 - 4x + 12}{(x - 3)^2}$$



THE QUOTIENT RULE

Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x - 3)^2}$$

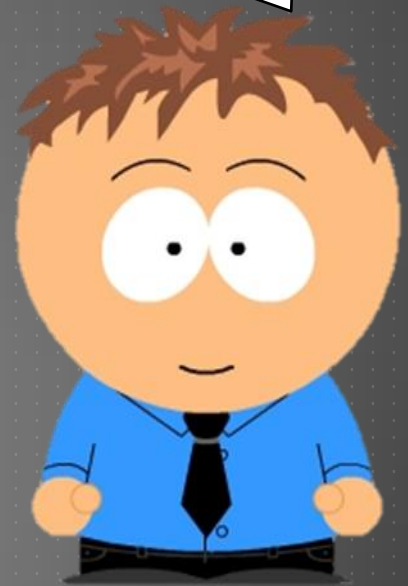


THE QUOTIENT RULE

Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x - 3)^2}$$

Again, label u and v, and find the derivatives $\frac{du}{dx}$ and $\frac{dv}{dx}$



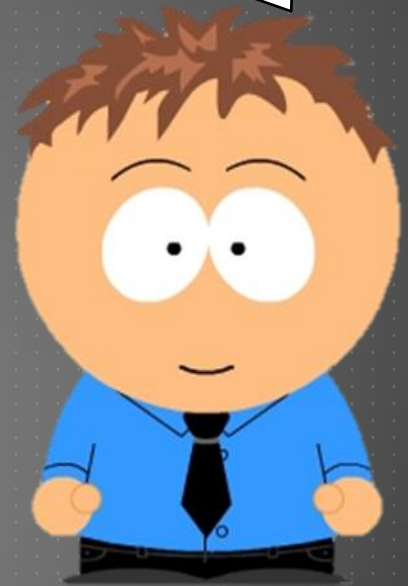
THE QUOTIENT RULE

Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x-3)^2}$$

$$u = x^2 - 4x + 12$$

Again, label u and v, and find the derivatives $\frac{du}{dx}$ and $\frac{dv}{dx}$



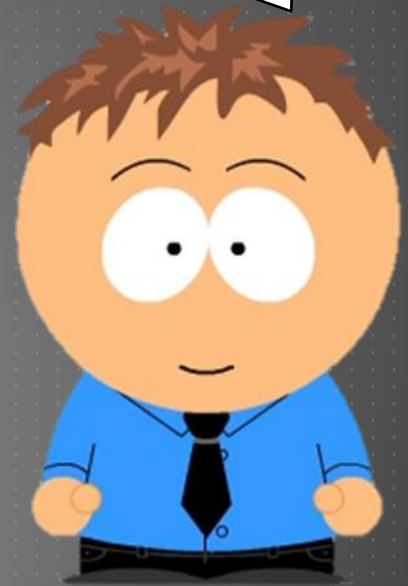
THE QUOTIENT RULE

Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x - 3)^2}$$

$$u = x^2 - 4x + 12 \qquad v = (x - 3)^2$$

Again, label u and v , and find the derivatives $\frac{du}{dx}$ and $\frac{dv}{dx}$



THE QUOTIENT RULE

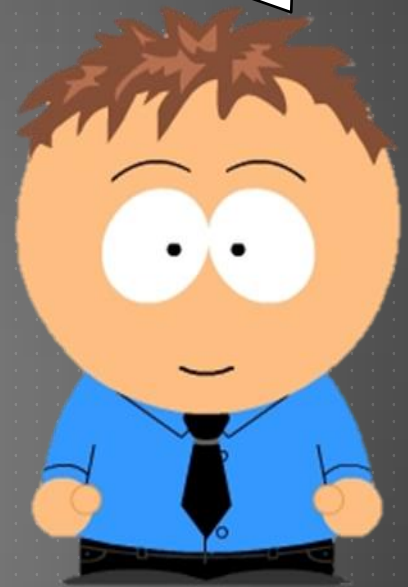
Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x - 3)^2}$$

$$u = x^2 - 4x + 12 \qquad v = (x - 3)^2$$

$$\frac{du}{dx} = 2x - 4$$

Again, label u and v, and find the derivatives $\frac{du}{dx}$ and $\frac{dv}{dx}$



THE QUOTIENT RULE

Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x - 3)^2}$$

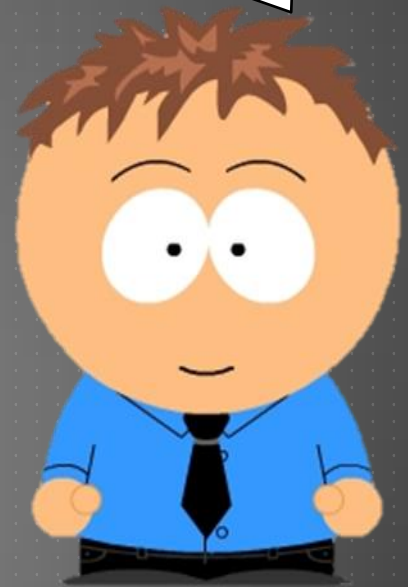
$$u = x^2 - 4x + 12$$

$$v = (x - 3)^2$$

$$\frac{du}{dx} = 2x - 4$$

$$\frac{dv}{dx} = 2(x - 3)$$

Again, label u and v, and find the derivatives $\frac{du}{dx}$ and $\frac{dv}{dx}$



THE QUOTIENT RULE

Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x - 3)^2}$$

$$u = x^2 - 4x + 12$$

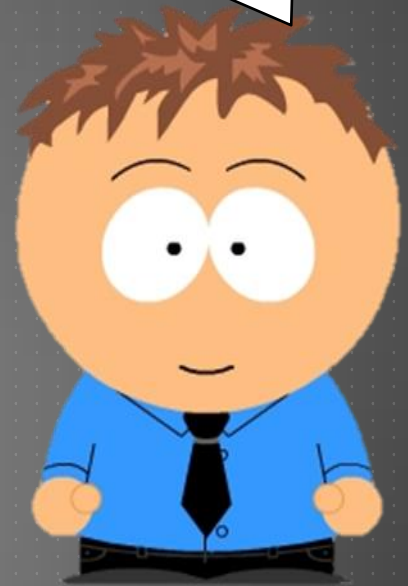
$$v = (x - 3)^2$$

$$\frac{du}{dx} = 2x - 4$$

$$\frac{dv}{dx} = 2(x - 3)$$

This time we substitute into the formula:

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$



THE QUOTIENT RULE

Differentiating when one function is divided by another

$$f(x) = \frac{x^2 - 4x + 12}{(x-3)^2}$$

$$u = x^2 - 4x + 12$$

$$v = (x-3)^2$$

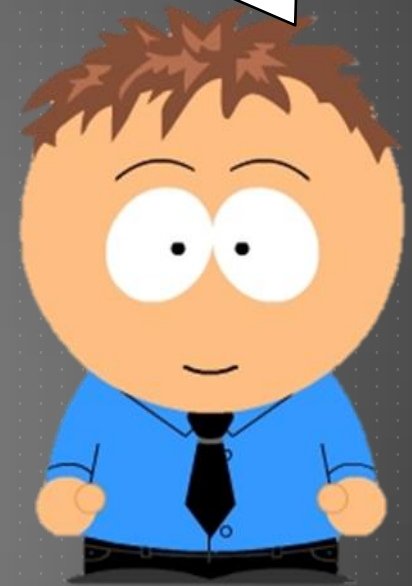
$$\frac{du}{dx} = 2x - 4$$

$$\frac{dv}{dx} = 2(x-3)$$

$$f'(x) = \frac{(x-3)^2 \times (2x-4) - (x^2-4x+12) \times 2(x-3)}{[(x-3)^2]^2}$$

This time we substitute into the formula:

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$



THE QUOTIENT RULE

Differentiating when one function is divided by another

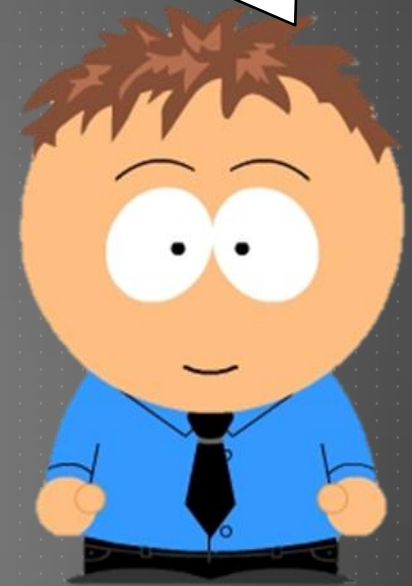
$$f(x) = \frac{x^2 - 4x + 12}{(x-3)^2}$$

$$u = x^2 - 4x + 12 \qquad v = (x-3)^2$$

$$\frac{du}{dx} = 2x - 4 \qquad \frac{dv}{dx} = 2(x-3)$$

$$f'(x) = \frac{(x-3)^2 \times (2x-4) - (x^2-4x+12) \times 2(x-3)}{[(x-3)^2]^2}$$

Again this needs a bit of tidying up...you can start by taking a factor of $(x-3)$ from all the terms



THE QUOTIENT RULE

Differentiating when one function is divided by another

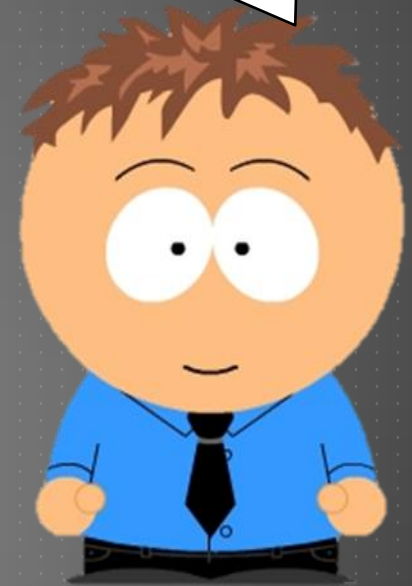
$$f(x) = \frac{x^2 - 4x + 12}{(x-3)^2}$$

$$u = x^2 - 4x + 12 \quad v = (x-3)^2$$

$$\frac{du}{dx} = 2x - 4 \quad \frac{dv}{dx} = 2(x-3)$$

$$\begin{aligned} f'(x) &= \frac{(x-3)^2 \times (2x-4) - (x^2-4x+12) \times 2(x-3)}{[(x-3)^2]^2} \\ &= \frac{(x-3) \times (2x-4) - (x^2-4x+12) \times 2}{(x-3)^3} \end{aligned}$$

Again this needs a bit of tidying up...you can start by taking a factor of $(x-3)$ from all the terms



THE QUOTIENT RULE

Differentiating when one function is divided by another

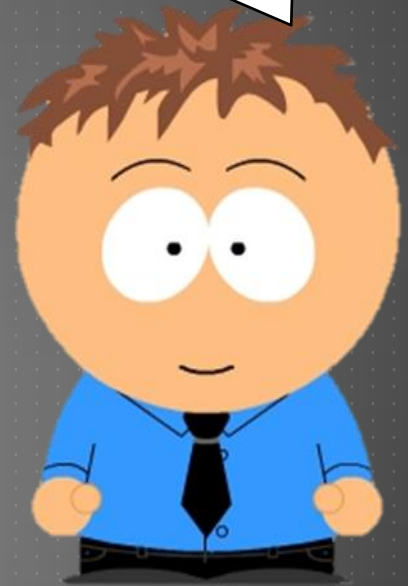
$$f(x) = \frac{x^2 - 4x + 12}{(x-3)^2}$$

$$u = x^2 - 4x + 12 \quad v = (x-3)^2$$

$$\frac{du}{dx} = 2x - 4 \quad \frac{dv}{dx} = 2(x-3)$$

$$\begin{aligned} f'(x) &= \frac{(x-3)^2 \times (2x-4) - (x^2-4x+12) \times 2(x-3)}{[(x-3)^2]^2} \\ &= \frac{(x-3) \times (2x-4) - (x^2-4x+12) \times 2}{(x-3)^3} \end{aligned}$$

And it should eventually simplify further to the result...



THE QUOTIENT RULE

Differentiating when one function is divided by another

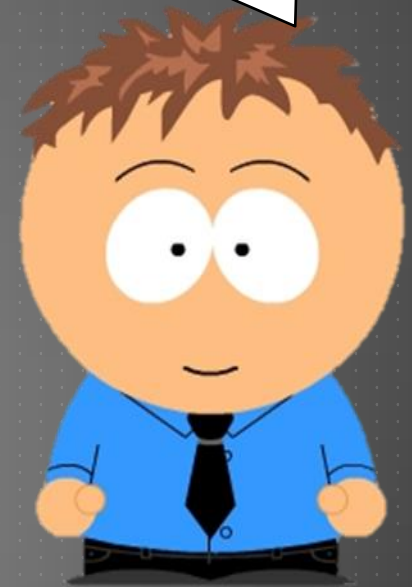
$$f(x) = \frac{x^2 - 4x + 12}{(x-3)^2}$$

$$u = x^2 - 4x + 12 \quad v = (x-3)^2$$

$$\frac{du}{dx} = 2x - 4 \quad \frac{dv}{dx} = 2(x-3)$$

$$\begin{aligned} f'(x) &= \frac{(x-3)^2 \times (2x-4) - (x^2-4x+12) \times 2(x-3)}{[(x-3)^2]^2} \\ &= \frac{(x-3) \times (2x-4) - (x^2-4x+12) \times 2}{(x-3)^3} = -\frac{2(x+6)}{(x-3)^3} \end{aligned}$$

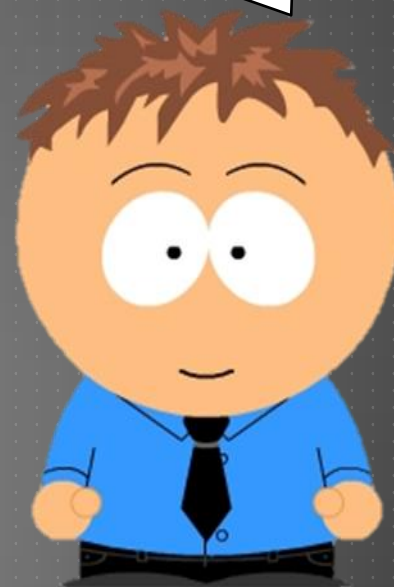
And it should eventually simplify further to the result...



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

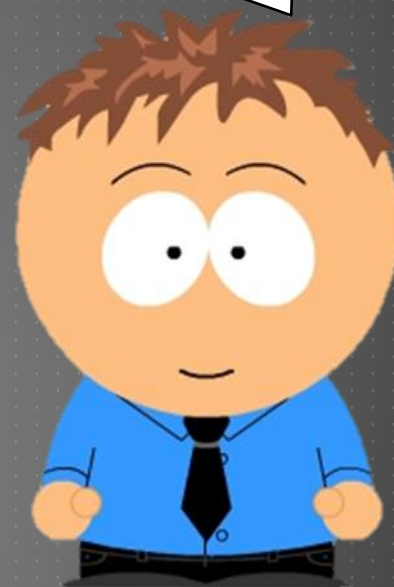
Previously we saw how we could try to reverse the chain rule, and integrate functions like $(2x - 3)^4$ *by inspection*



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

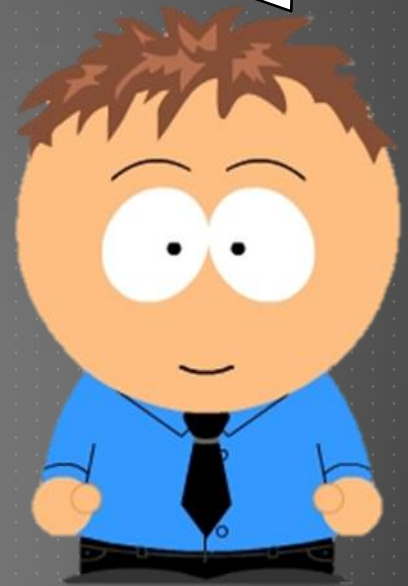
This idea can be extended to
some other products and
quotients, using a method called
substitution



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

It works if one part of the equation can be differentiated and used to cancel the other

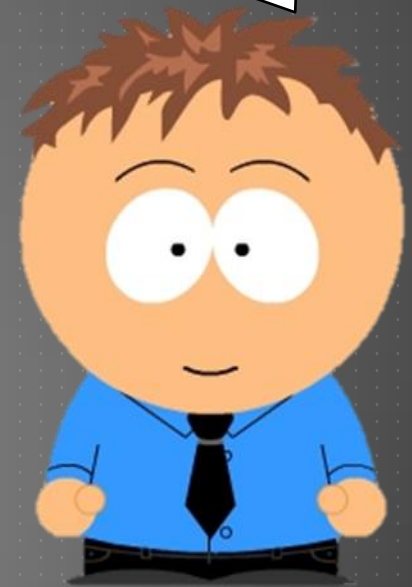


INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Take the example
 $y = (2x + 1)e^{x^2+x-1}$

Find $\int y \, dx$



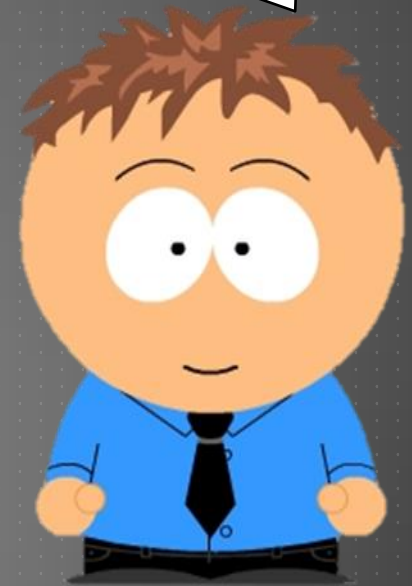
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

Take the example
 $y = (2x + 1)e^{x^2+x-1}$

Find $\int y dx$

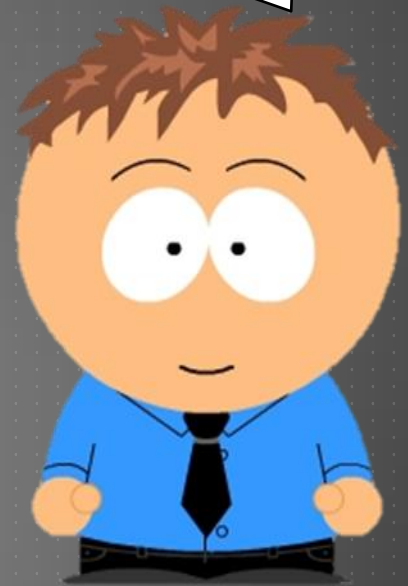


INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

The *inner* part of the composite function e^{x^2+x-1} Differentiates to give the second function, $2x + 1$

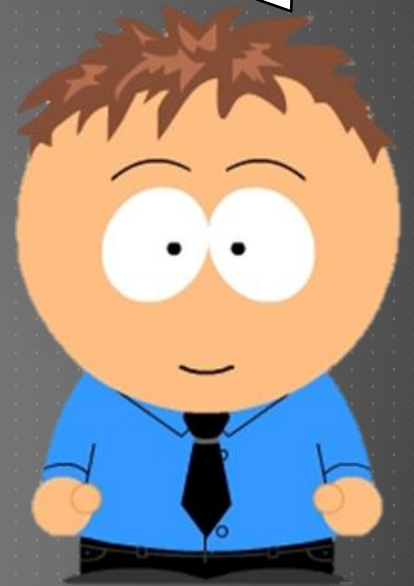


INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

First we make a *substitution* for the inner function



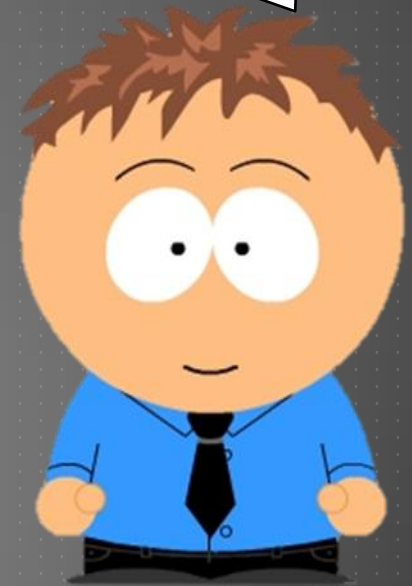
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2 + x - 1} dx$$

$$\text{Let } u = x^2 + x - 1$$

First we make a *substitution*, for the inner function



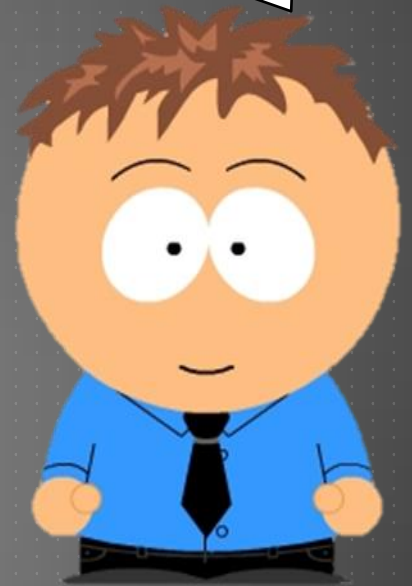
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

And differentiate it with respect
to x



INTEGRATION BY SUBSTITUTION

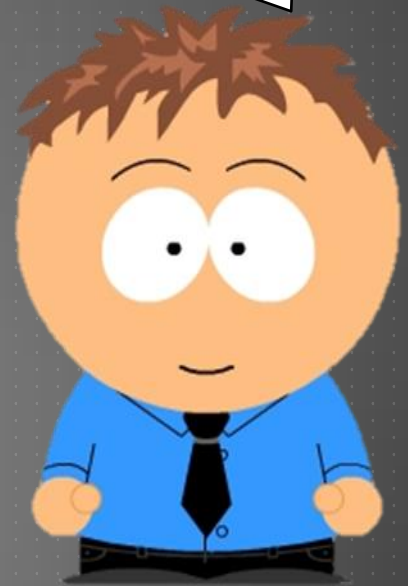
Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1$$

And differentiate it with respect
to x



INTEGRATION BY SUBSTITUTION

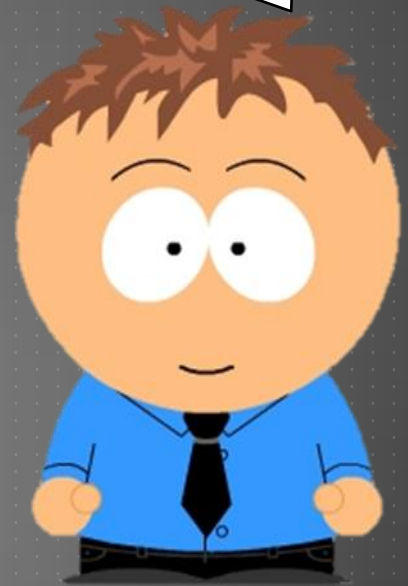
Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1$$

If we now rewrite the integral with our substitution it looks like...



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

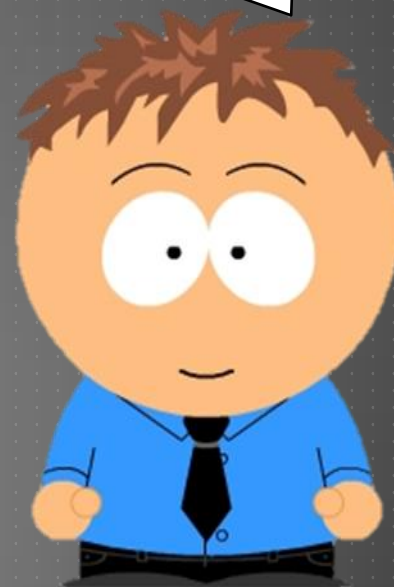
$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1$$

$$= \int (2x + 1)e^u dx$$

If we now rewrite the integral with our substitution it looks like...



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

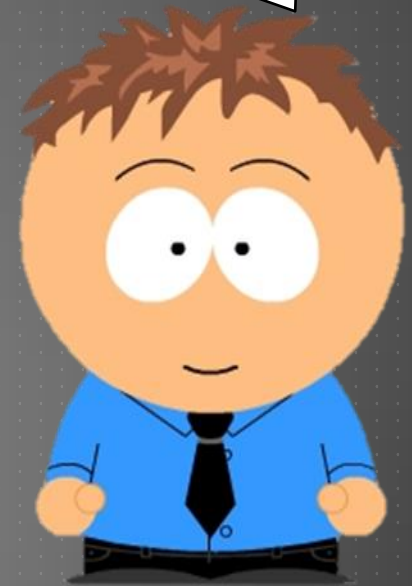
$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1$$

$$= \int (2x + 1)e^u dx$$

We now have the variable u in our equation, so we no longer have to be integrating with respect to x



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

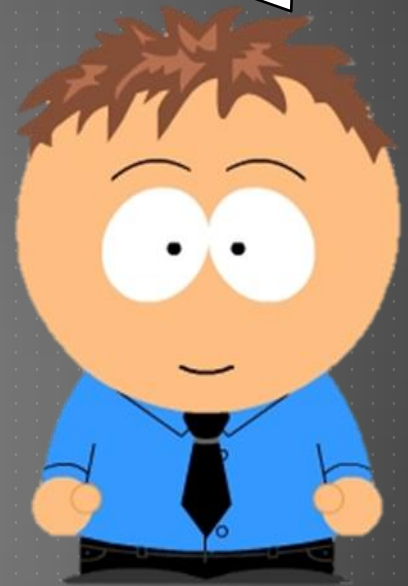
$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1$$

$$= \int (2x + 1)e^u dx$$

We can however find du in terms of dx , by using knowledge of the derivative $\frac{du}{dx}$



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

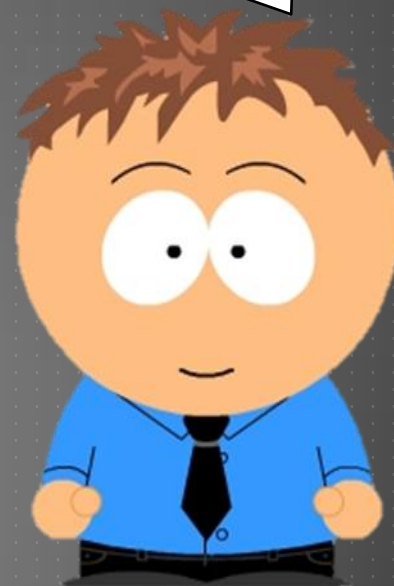
$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1$$

$$= \int (2x + 1)e^u dx$$

Rearrange to find dx



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

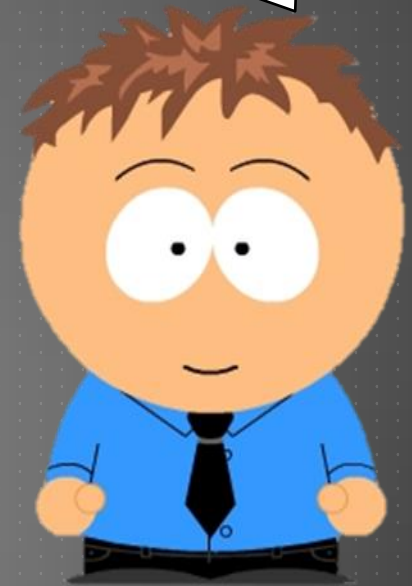
$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

$$= \int (2x + 1)e^u dx$$

Rearrange to find dx



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

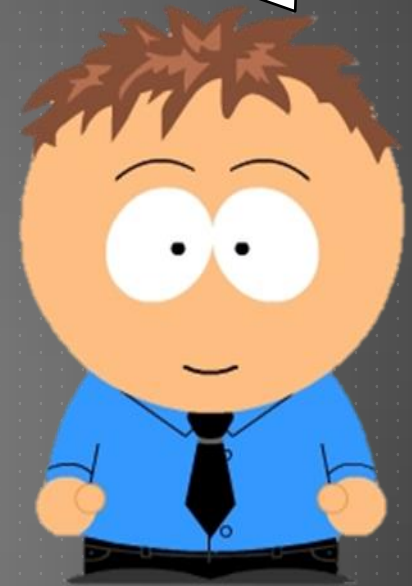
$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

$$= \int (2x + 1)e^u dx$$

And replace back in the equation



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

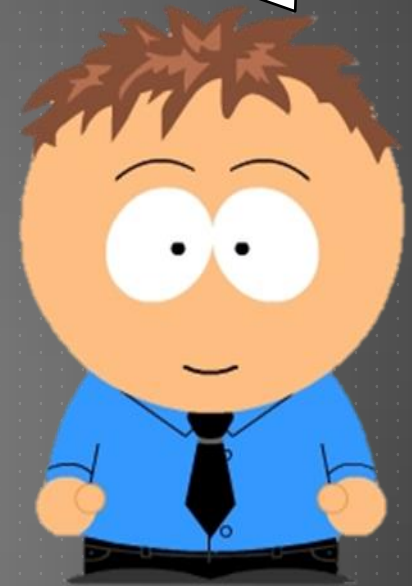
$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

$$= \int (2x + 1)e^u dx$$

$$= \int (2x + 1)e^u \frac{du}{2x+1}$$

And replace back in the equation



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

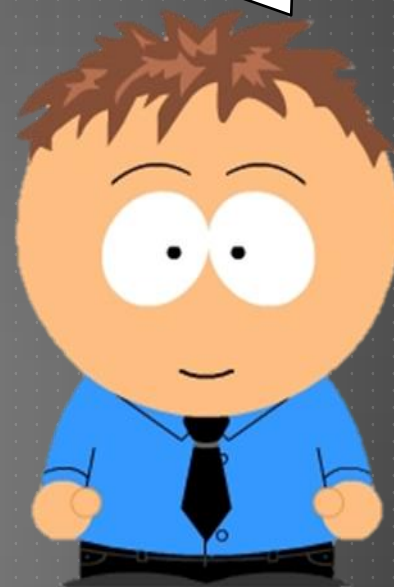
$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

$$= \int (2x + 1)e^u dx$$

$$= \int (2x + 1)e^u \frac{du}{2x+1}$$

We now have the situation where two parts of our expression cancel out, leaving a much simpler integral



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

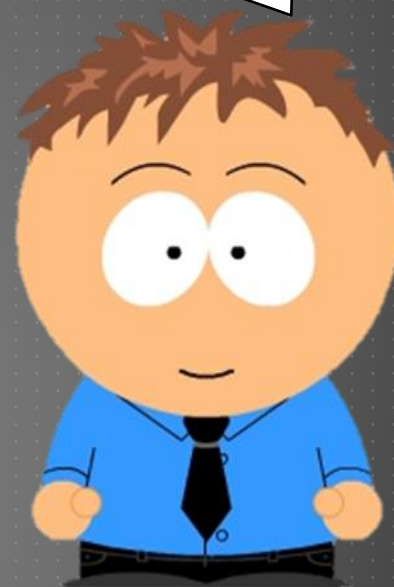
$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

$$= \int (2x + 1)e^u dx$$

$$= \int (2x + 1)e^u \frac{du}{2x+1}$$

$$= \int e^u du$$

We now have the situation where two parts of our expression cancel out, leaving a much simpler integral



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

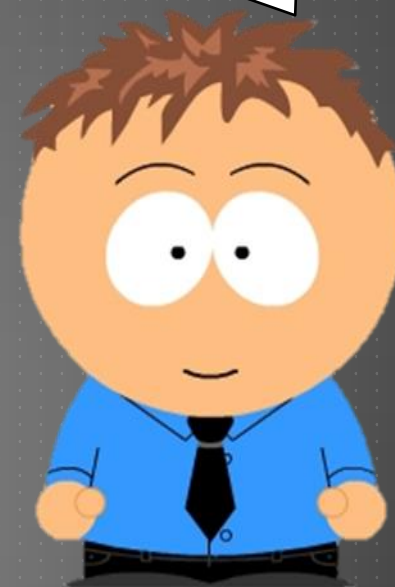
$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

$$= \int (2x + 1)e^u dx$$

$$= \int (2x + 1)e^u \frac{du}{2x+1}$$

$$= \int e^u du$$

The final step is to integrate, and substitute back in our expression for u



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

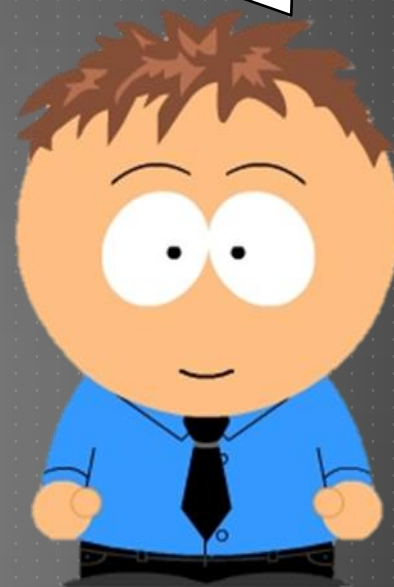
$$= \int (2x + 1)e^u dx$$

$$= \int (2x + 1)e^u \frac{du}{2x+1}$$

$$= \int e^u du$$

$$= e^u + c$$

We now have the situation where two parts of our expression cancel out, leaving a much simpler integral



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

$$\int (2x + 1)e^{x^2+x-1} dx$$

$$\text{Let } u = x^2 + x - 1$$

$$\frac{du}{dx} = 2x + 1 \quad \Rightarrow \quad \frac{du}{2x+1} = dx$$

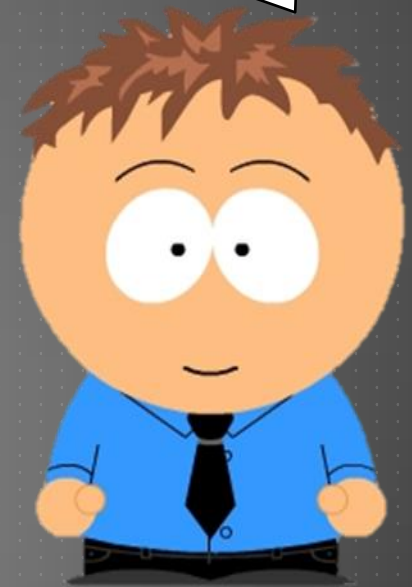
$$= \int (2x + 1)e^u dx$$

$$= \int (2x + 1)e^u \frac{du}{2x+1}$$

$$= \int e^u du$$

$$= e^u + c = e^{x^2+x-1} + c$$

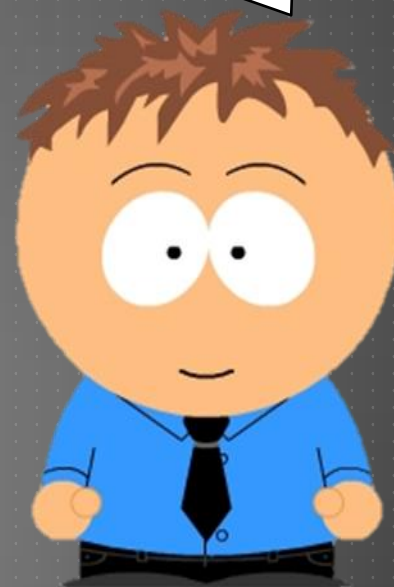
We now have the situation where two parts of our expression cancel out, leaving a much simpler integral



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

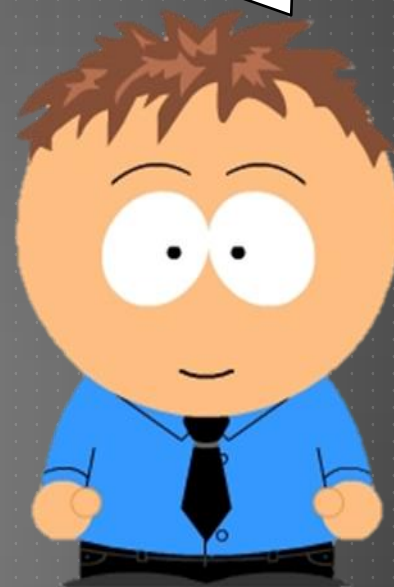
The trick to this method is spotting the substitution you can make to simplify the expression



INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Sometimes it will be obvious, but
if in doubt usually you can follow
the following rule of thumb



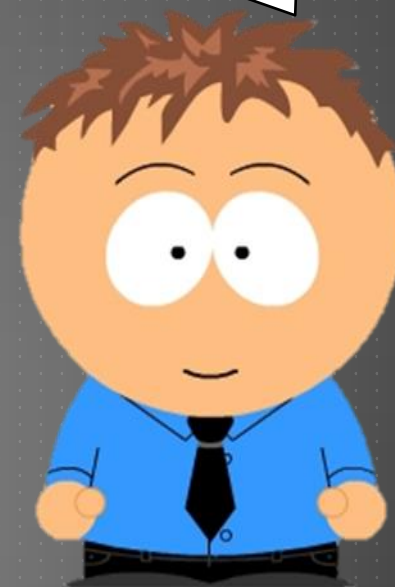
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	
$\int \frac{x^2}{5x^3 - 1} dx$	
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

Sometimes it will be obvious...



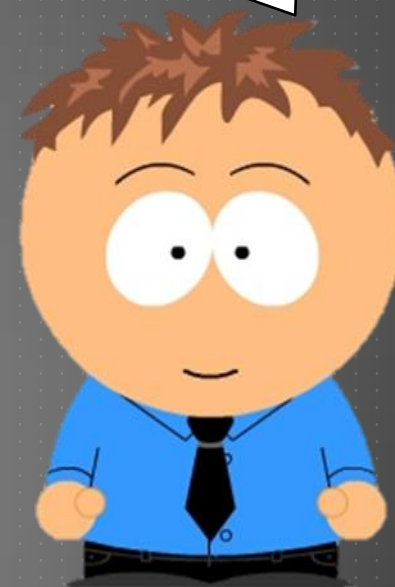
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	
$\int \frac{x^2}{5x^3 - 1} dx$	
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

If in doubt you can usually follow the general rule:



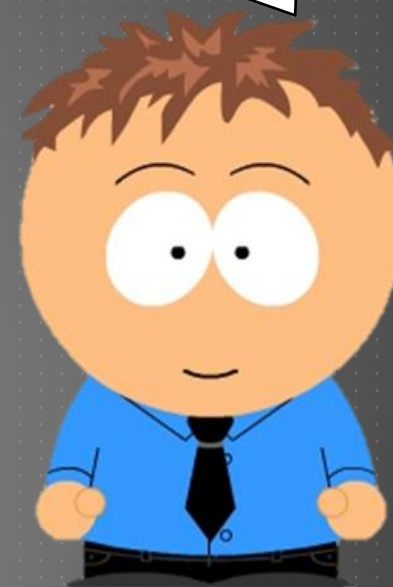
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	
$\int \frac{x^2}{5x^3 - 1} dx$	
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

1. If you have a composite function, choose the *inner* part



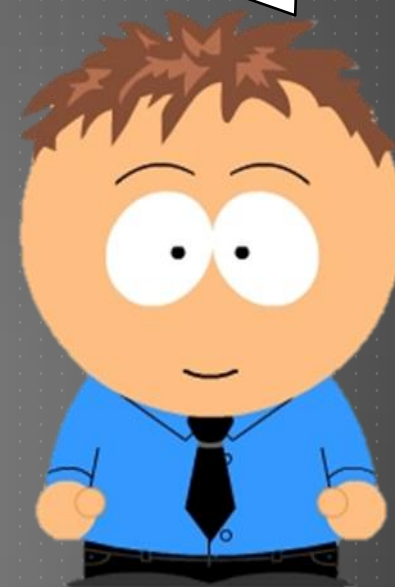
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	
$\int \frac{x^2}{5x^3 - 1} dx$	
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

1. If you have a composite function, choose the *inner* part



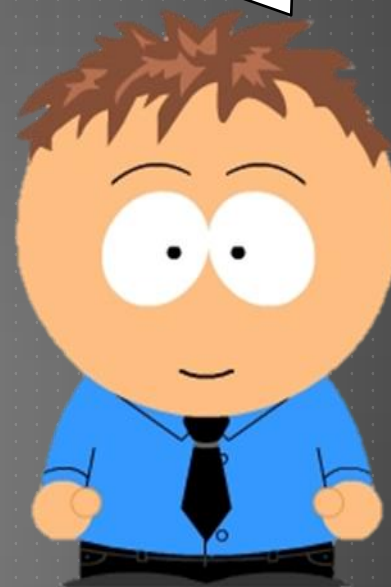
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	$u = x^2 + 3$
$\int \frac{x^2}{5x^3 - 1} dx$	
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

1. If you have a composite function, choose the *inner* part



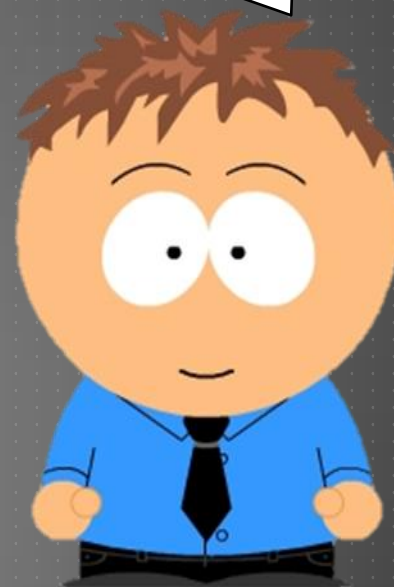
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	$u = x^2 + 3$
$\int \frac{x^2}{5x^3 - 1} dx$	
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

2. If you have a fraction, choose the denominator (expression on the bottom)



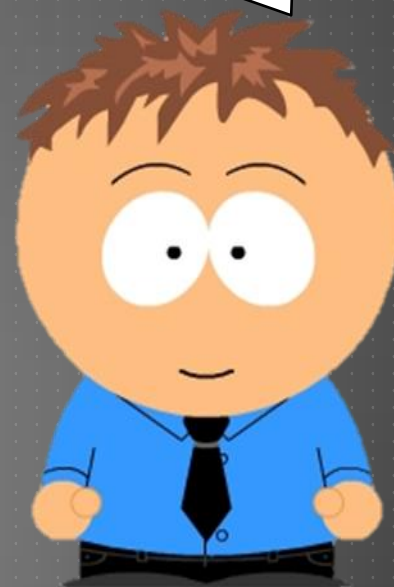
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	$u = x^2 + 3$
$\int \frac{x^2}{5x^3 - 1} dx$	
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

2. If you have a fraction, choose the denominator (expression on the bottom)



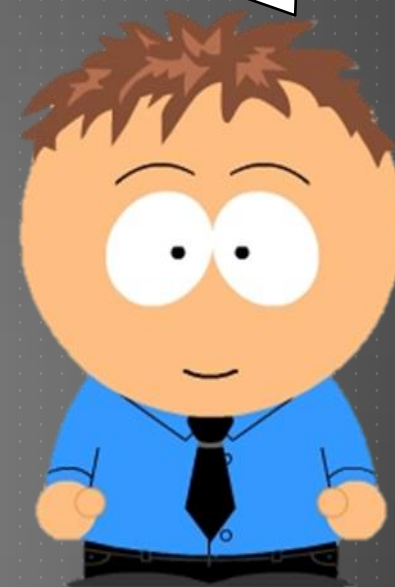
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	$u = x^2 + 3$
$\int \frac{x^2}{5x^3 - 1} dx$	$u = 5x^3 - 1$
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

2. If you have a fraction, choose the denominator (expression on the bottom)



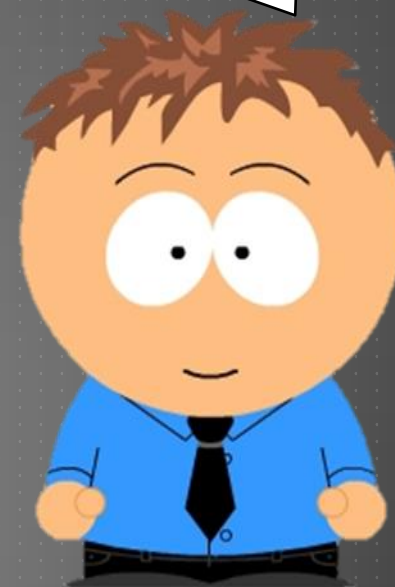
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	$u = x^2 + 3$
$\int \frac{x^2}{5x^3 - 1} dx$	$u = 5x^3 - 1$
$\int 3xe^{x^2} dx$	
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

What will the next two be?...



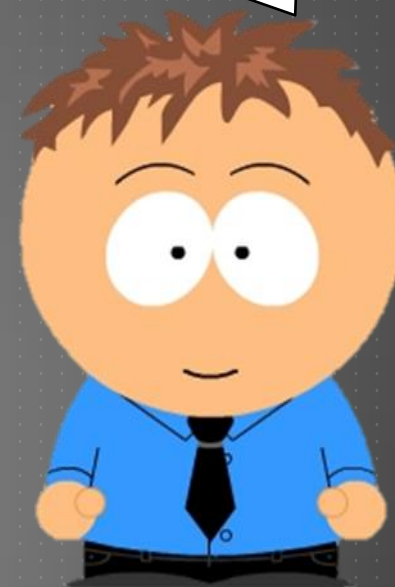
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	$u = x^2 + 3$
$\int \frac{x^2}{5x^3 - 1} dx$	$u = 5x^3 - 1$
$\int 3xe^{x^2} dx$	$u = x^2$
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	

What will the next two be?...



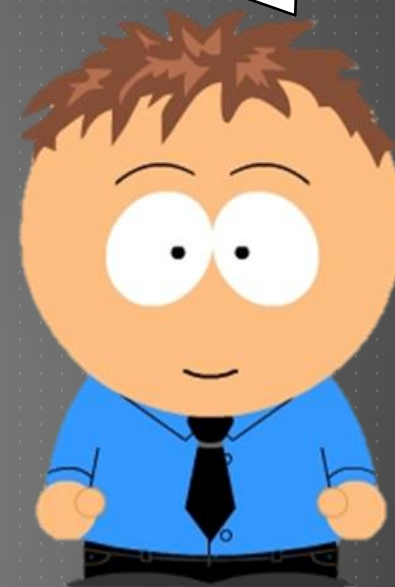
INTEGRATION BY SUBSTITUTION

Integrating more complex expressions

Examples

Expression	Substitution
$\int x(x^2 + 3)^3 dx$	$u = x^2 + 3$
$\int \frac{x^2}{5x^3 - 1} dx$	$u = 5x^3 - 1$
$\int 3xe^{x^2} dx$	$u = x^2$
$\int \frac{e^x}{\sqrt{e^x + 1}} dx$	$u = e^x + 1$

What will the next two be?...



EXAMPLES

1.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $f'(x)$

Find $\int f(x) dx$

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

Find $\int_0^1 g(x) dx$

SOLUTIONS

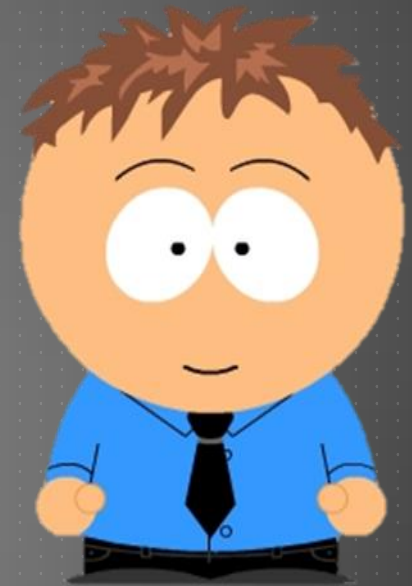
1.

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

When differentiating a fraction, you can use the **quotient rule**



SOLUTIONS

1.

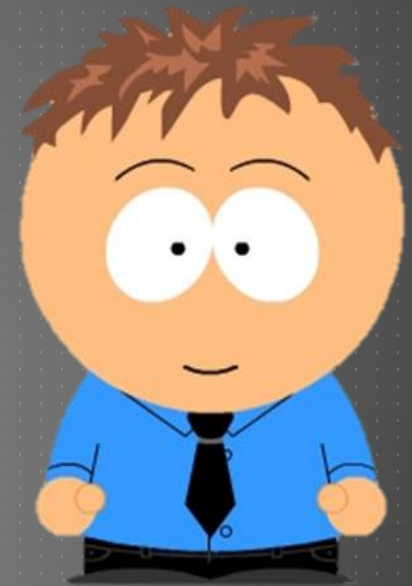
Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Formula book:

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$



SOLUTIONS

1.

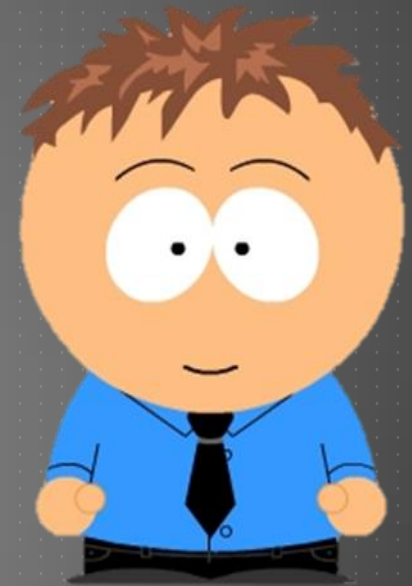
$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array}$$

Formula book:

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$



SOLUTIONS

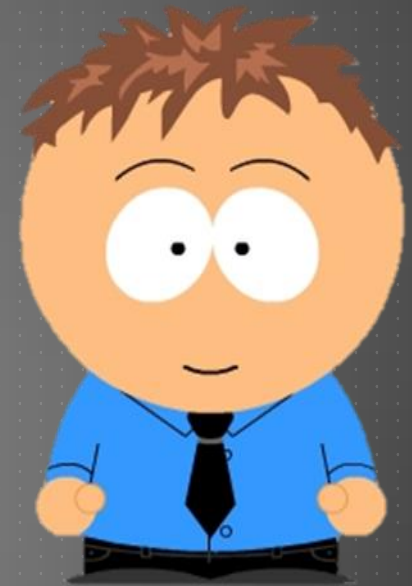
$$f(x) = \frac{\sin(x)}{\cos(x)}$$

1.

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array}$$

Calculate the derivatives of u and v



SOLUTIONS

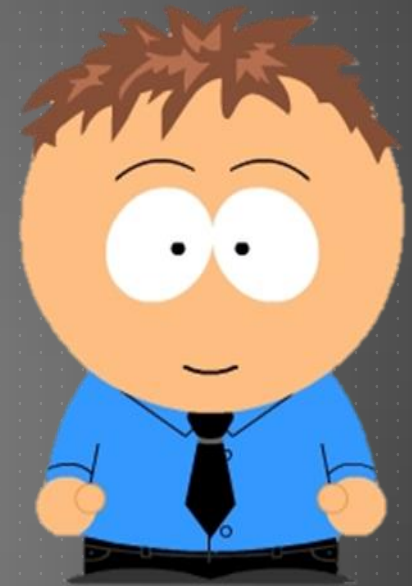
$$f(x) = \frac{\sin(x)}{\cos(x)}$$

1.

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array} \quad \begin{array}{l} \frac{du}{dx} = \cos(x) \\ \frac{dv}{dx} = -\sin(x) \end{array}$$

Calculate the derivatives of u and v



SOLUTIONS

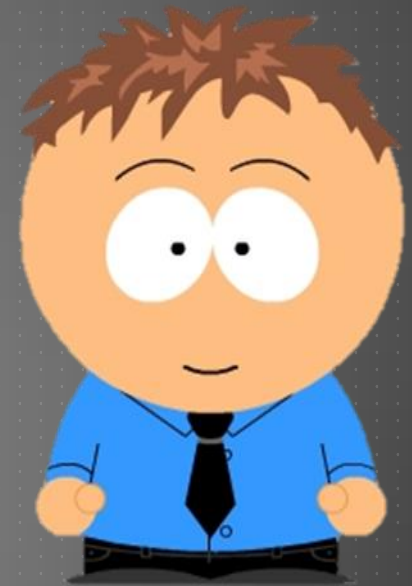
I.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array} \quad \begin{array}{l} \frac{du}{dx} = \cos(x) \\ \frac{dv}{dx} = -\sin(x) \end{array}$$

Substitute all the values back into the formula



SOLUTIONS

1.

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array}$$

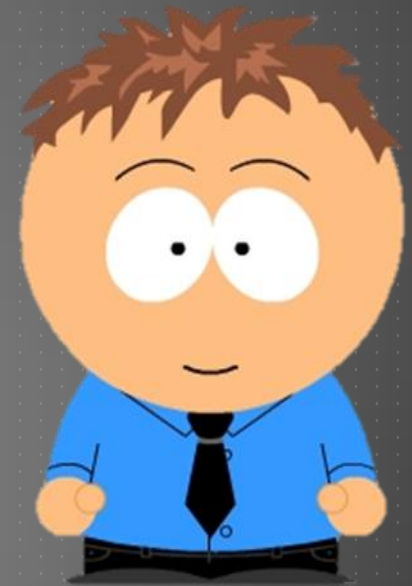
$$\frac{du}{dx} = \cos(x)$$

$$\frac{dv}{dx} = -\sin(x)$$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot \sin(x)}{(\cos(x))^2}$$

Formula book:

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$



SOLUTIONS

1.

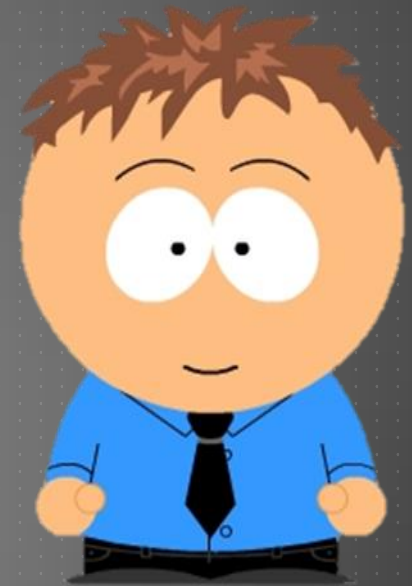
$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array} \quad \begin{array}{l} \frac{du}{dx} = \cos(x) \\ \frac{dv}{dx} = -\sin(x) \end{array}$$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot \sin(x)}{(\cos(x))^2}$$

Simplify where possible



SOLUTIONS

1.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

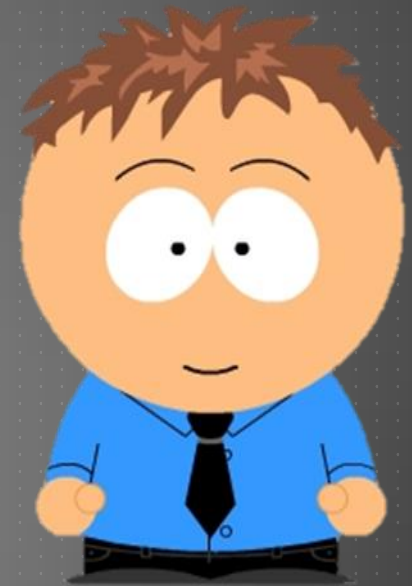
Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array} \quad \frac{du}{dx} = \cos(x)$$

$$\frac{dv}{dx} = -\sin(x)$$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot \sin(x)}{(\cos(x))^2}$$

Remember that
 $\cos(x) \times \cos(x) = \cos^2 x$



SOLUTIONS

1.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $f'(x)$

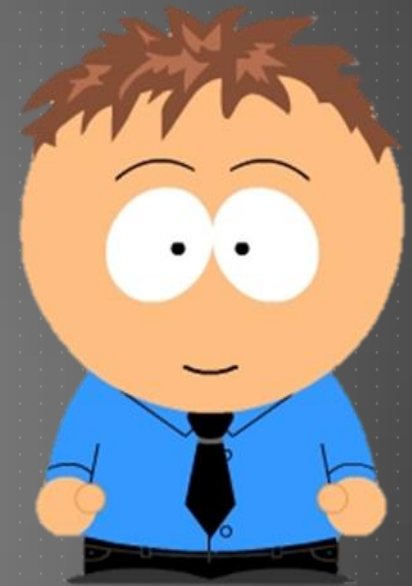
$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array} \quad \frac{du}{dx} = \cos(x)$$

$$\frac{dv}{dx} = -\sin(x)$$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot -\sin(x)}{(\cos(x))^2}$$

$$f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

Remember that
 $\cos(x) \times \cos(x) = \cos^2 x$



SOLUTIONS

1.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $f'(x)$

$$f(x) = \frac{\sin(x)}{\cos(x)} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array} \quad \frac{du}{dx} = \cos(x)$$

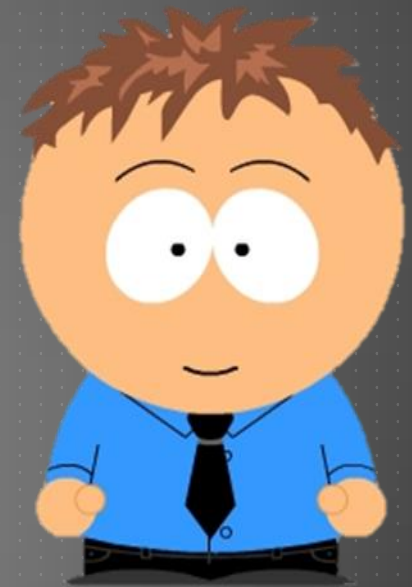
$$\frac{dv}{dx} = -\sin(x)$$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot -\sin(x)}{(\cos(x))^2}$$

$$f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

And that $\sin^2 x$ and $\cos^2 x$ are related by:

$$\sin^2 x + \cos^2 x = 1$$



SOLUTIONS

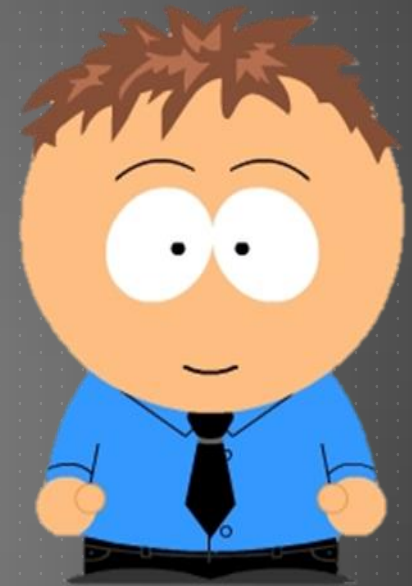
1.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx$$

For harder integrals you will have to use the **substitution method**



SOLUTIONS

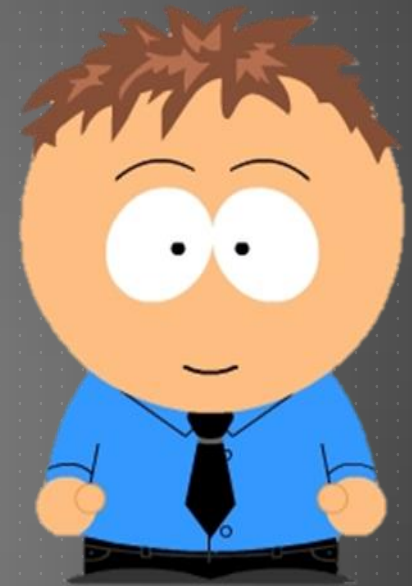
1.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx$$

Choose a suitable substitution
and differentiate to find how
to change **dx** to **du**



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

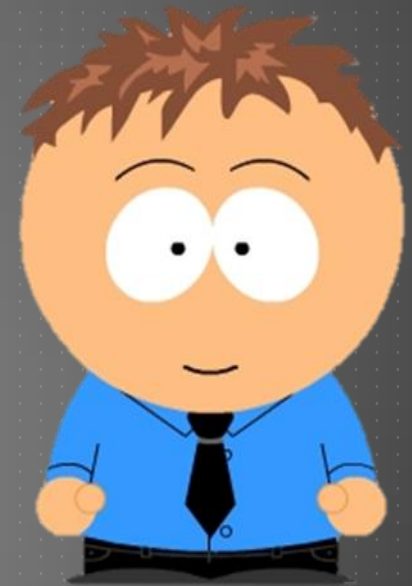
1.

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx$$

$$u = \cos(x)$$

Choose a suitable substitution
and differentiate to find how
to change **dx** to **du**



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

1.

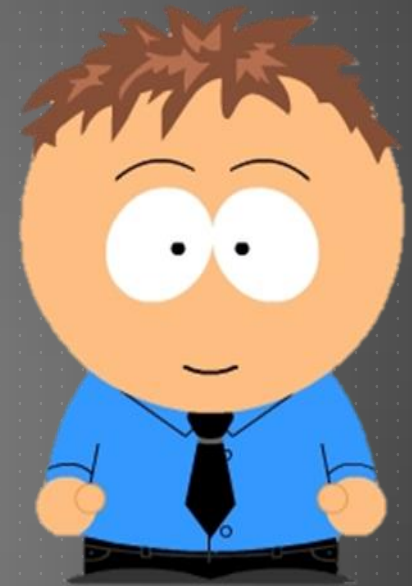
Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx$$

$$u = \cos(x)$$

$$\frac{du}{dx} = -\sin(x)$$

Choose a suitable substitution
and differentiate to find how
to change **dx** to **du**



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

1.

Find $\int f(x) dx$

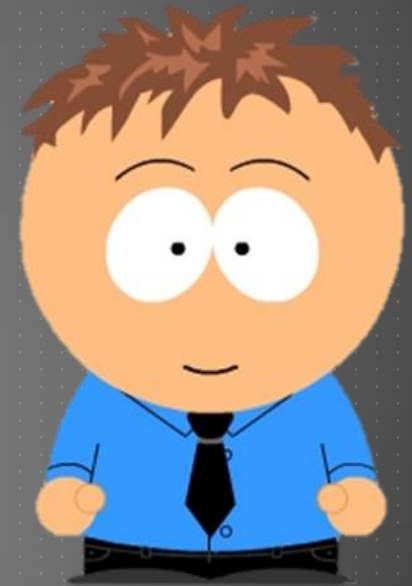
$$\int \frac{\sin(x)}{\cos(x)} dx$$

$$u = \cos(x)$$

$$\frac{du}{dx} = -\sin(x)$$

$$\frac{du}{-\sin(x)} = dx$$

Choose a suitable substitution and differentiate to find how to change **dx** to **du**



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

1.

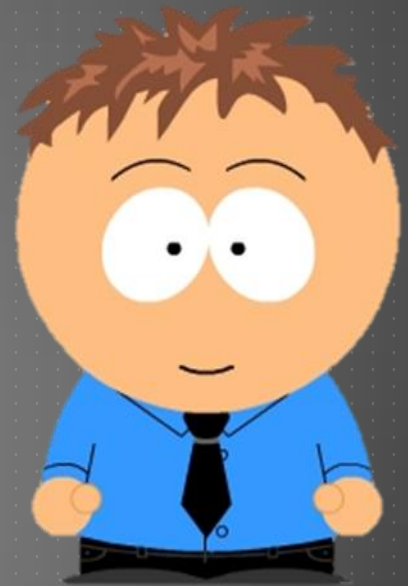
Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx$$

$$u = \cos(x)$$

$$\frac{du}{dx} = -\sin(x) \quad \frac{du}{-\sin(x)} = dx$$

Rewrite the integral



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

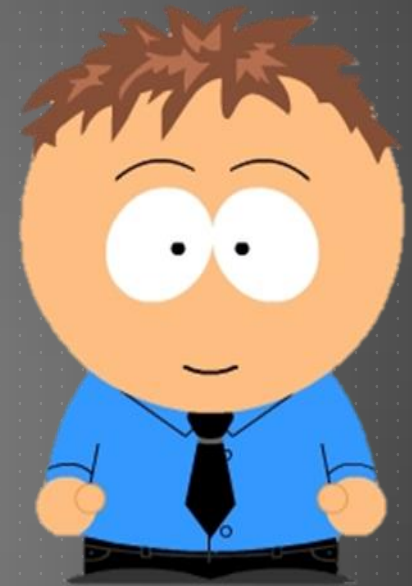
1.

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx \quad u = \cos(x) \quad \frac{du}{dx} = -\sin(x) \quad \frac{du}{-\sin(x)} = dx$$

$$\int \frac{\sin(x)}{\cos(x)} dx = \int \frac{\sin(x)}{u} \frac{du}{-\sin(x)}$$

Rewrite the integral



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

1.

Find $\int f(x) dx$

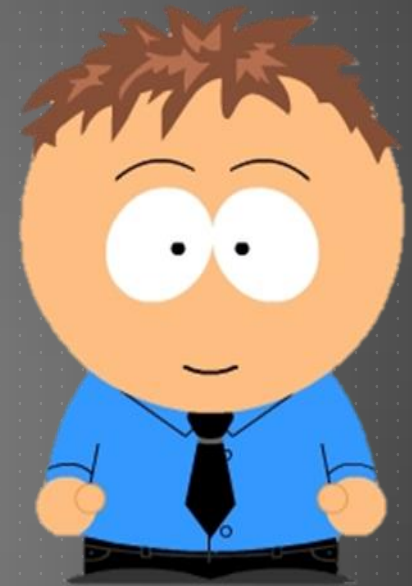
$$\int \frac{\sin(x)}{\cos(x)} dx$$

$$u = \cos(x)$$

$$\frac{du}{dx} = -\sin(x) \quad \frac{du}{-\sin(x)} = dx$$

$$\int \frac{\sin(x)}{\cos(x)} dx = \int \frac{\sin(x)}{u} \frac{du}{-\sin(x)}$$

Simplify and solve



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

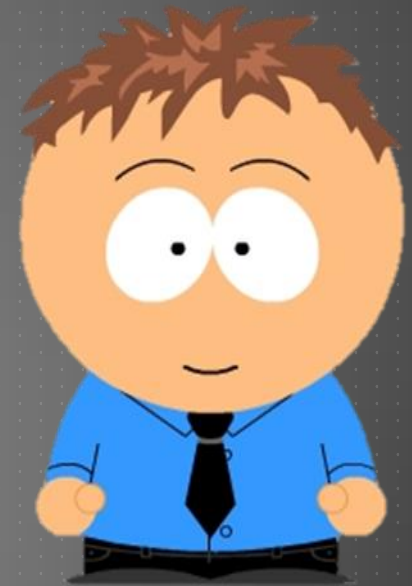
1.

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx \quad u = \cos(x) \quad \frac{du}{dx} = -\sin(x) \quad \frac{du}{-\sin(x)} = dx$$

$$\begin{aligned} \int \frac{\sin(x)}{\cos(x)} dx &= \int \frac{\sin(x)}{u} \frac{du}{-\sin(x)} \\ &= \int -\frac{1}{u} du \end{aligned}$$

Simplify and solve



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

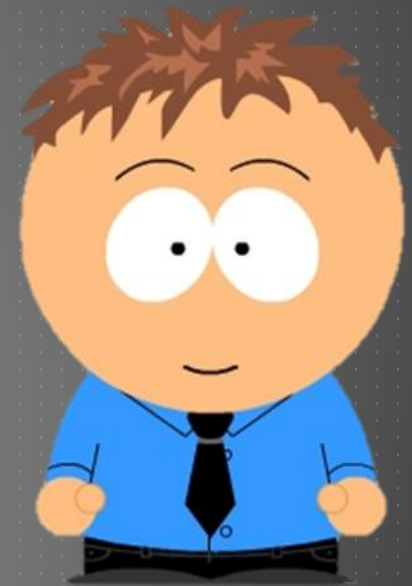
1.

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx \quad u = \cos(x) \quad \frac{du}{dx} = -\sin(x) \quad \frac{du}{-\sin(x)} = dx$$

$$\begin{aligned} \int \frac{\sin(x)}{\cos(x)} dx &= \int \frac{\sin(x)}{u} \frac{du}{-\sin(x)} \\ &= \int -\frac{1}{u} du = -\ln(u) + C \end{aligned}$$

Simplify and solve



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

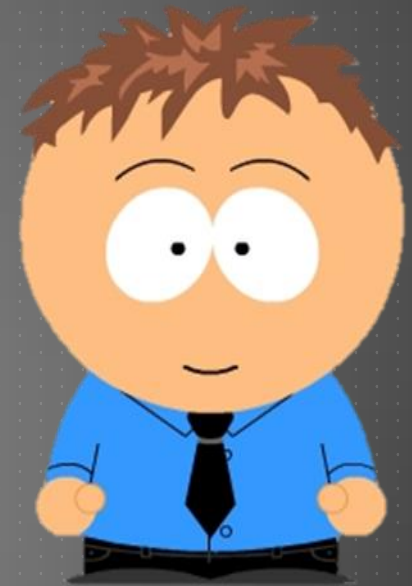
1.

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx \quad u = \cos(x) \quad \frac{du}{dx} = -\sin(x) \quad \frac{du}{-\sin(x)} = dx$$

$$\begin{aligned} \int \frac{\sin(x)}{\cos(x)} dx &= \int \frac{\sin(x)}{u} \frac{du}{-\sin(x)} \\ &= \int -\frac{1}{u} du = -\ln(u) + C \end{aligned}$$

Substitute x back in for u



SOLUTIONS

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

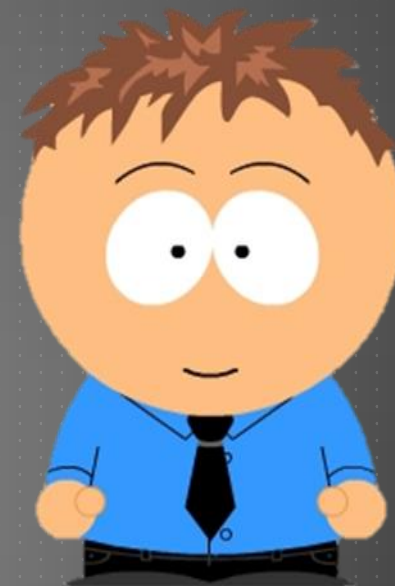
1.

Find $\int f(x) dx$

$$\int \frac{\sin(x)}{\cos(x)} dx \quad u = \cos(x) \quad \frac{du}{dx} = -\sin(x) \quad \frac{du}{-\sin(x)} = dx$$

$$\begin{aligned} \int \frac{\sin(x)}{\cos(x)} dx &= \int \frac{\sin(x)}{u} \frac{du}{-\sin(x)} \\ &= \int -\frac{1}{u} du = -\ln(u) + C \\ &= -\ln(\cos(x)) + C \end{aligned}$$

Substitute x back in for u



EXAMPLES

1.

$$f(x) = \frac{\sin(x)}{\cos(x)}$$

Find $f'(x)$

Find $\int f(x) dx$

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

Find $\int_0^1 g(x) dx$

SOLUTIONS

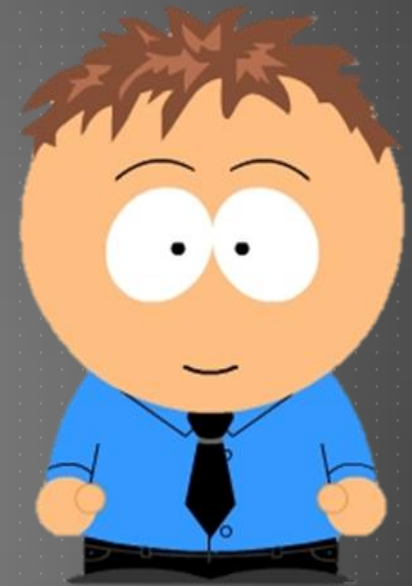
2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

Here is an example to use the **product rule**



SOLUTIONS

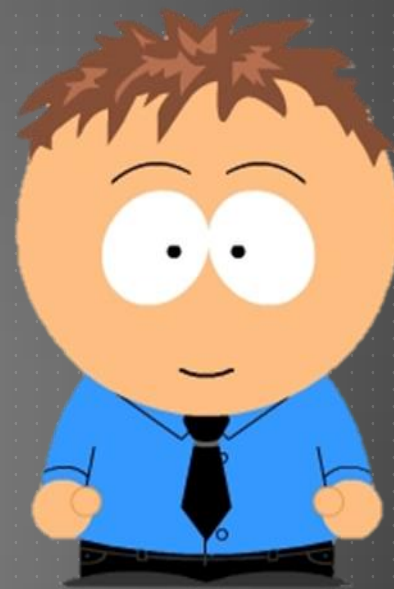
2.

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

$$g(x) = 5x^2\sqrt{1-x^3}$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



SOLUTIONS

I.

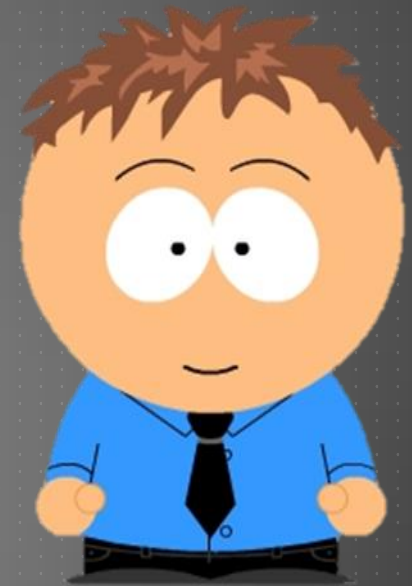
$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

$$u = 5x^2 \quad v = \sqrt{1-x^3}$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



SOLUTIONS

2.

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

$$g(x) = 5x^2\sqrt{1-x^3}$$

$$u = 5x^2 \quad v = \sqrt{1-x^3}$$

$$\frac{du}{dx} = 10x$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

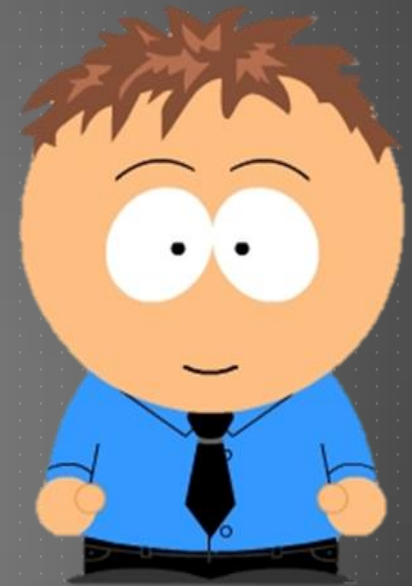
Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

$$u = 5x^2 \quad v = \sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = -3x^2 \times \frac{1}{2}(1-x^3)^{-\frac{1}{2}}$$

You might want to rewrite v using powers, to more easily differentiate using the chain rule



SOLUTIONS

2.

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

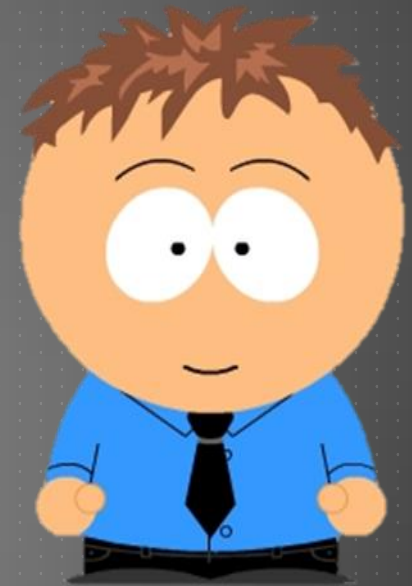
$$g(x) = 5x^2\sqrt{1-x^3}$$

$$u = 5x^2 \quad v = \sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = -3x^2 \times \frac{1}{2}(1-x^3)^{-\frac{1}{2}}$$

$$g'(x) =$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



SOLUTIONS

2.

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

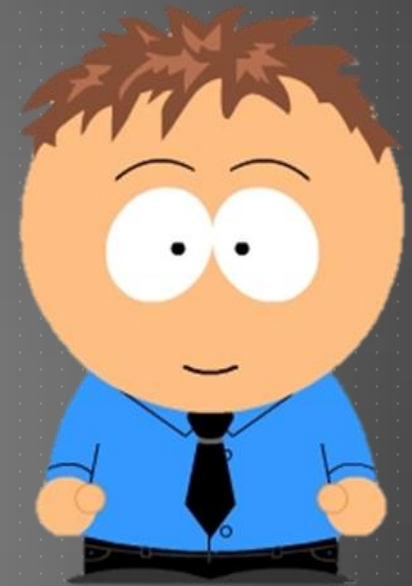
$$g(x) = 5x^2\sqrt{1-x^3}$$

$$u = 5x^2 \quad v = \sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = -3x^2 \times \frac{1}{2}(1-x^3)^{-\frac{1}{2}}$$

$$g'(x) = 5x^2 \times \frac{3x^2}{2\sqrt{1-x^3}}$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

$$u = 5x^2 \quad v = \sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = -3x^2 \times \frac{1}{2}(1-x^3)^{-\frac{1}{2}}$$

$$g'(x) = \frac{5x^2 \times -\frac{3x^2}{2\sqrt{1-x^3}} + \sqrt{1-x^3} \times 10x}{}$$

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

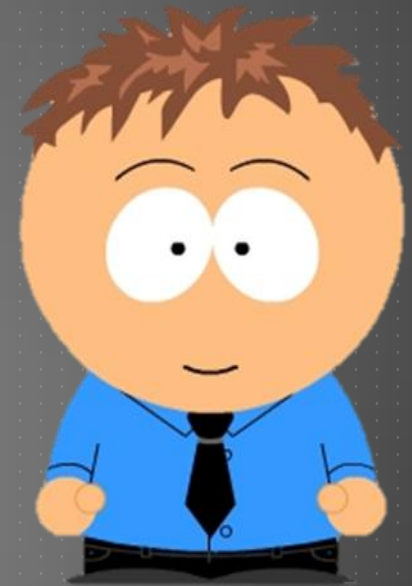
$$g(x) = 5x^2\sqrt{1-x^3}$$

$$u = 5x^2 \quad v = \sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = -3x^2 \times \frac{1}{2}(1-x^3)^{-\frac{1}{2}}$$

$$g'(x) = 5x^2 \times \frac{3x^2}{2\sqrt{1-x^3}} + \sqrt{1-x^3} \times 10x$$

This can be further simplified.
It is a little bit tricky, but by
this stage you will have gained
the majority of the marks



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

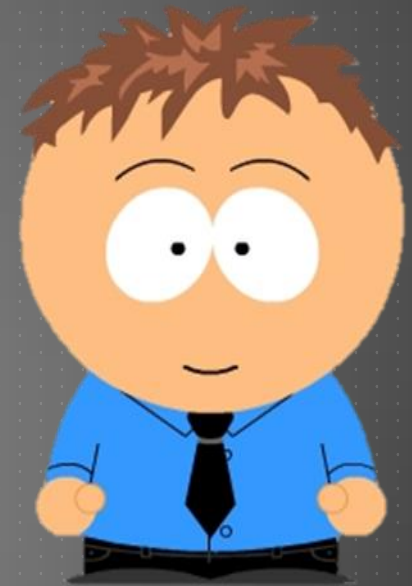
$$u = 5x^2 \quad v = \sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = -3x^2 \times \frac{1}{2}(1-x^3)^{-\frac{1}{2}}$$

$$g'(x) = \frac{5x^2 \times -\frac{3x^2}{2\sqrt{1-x^3}} + \sqrt{1-x^3} \times 10x}{2\sqrt{1-x^3}}$$

$$= -\frac{15x^4}{2\sqrt{1-x^3}} + \frac{20x(1-x^3)}{2\sqrt{1-x^3}}$$

Writing over common denominator helps



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $g'(x)$

$$g(x) = 5x^2\sqrt{1-x^3}$$

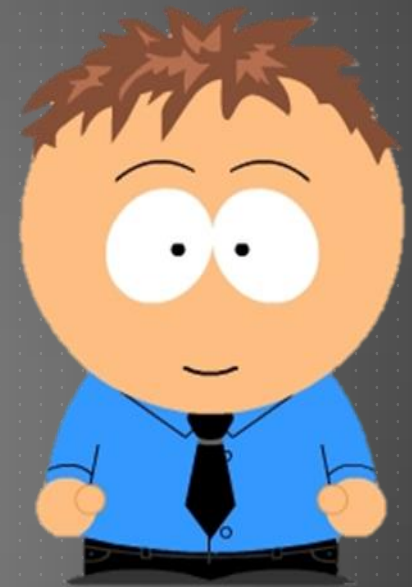
$$u = 5x^2 \quad v = \sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = -3x^2 \times \frac{1}{2}(1-x^3)^{-\frac{1}{2}}$$

$$g'(x) = \frac{5x^2 \times -\frac{3x^2}{2\sqrt{1-x^3}} + \sqrt{1-x^3} \times 10x}{}$$

$$= -\frac{15x^4}{2\sqrt{1-x^3}} + \frac{20x(1-x^3)}{2\sqrt{1-x^3}} = \frac{5x(4-7x^3)}{2\sqrt{1-x^3}}$$

And the final result (with a few steps skipped), can be seen below



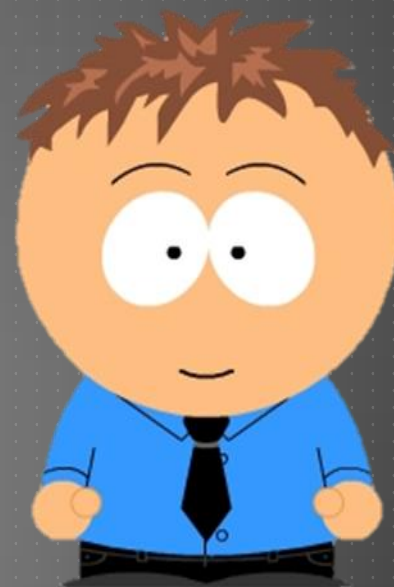
SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$



SOLUTIONS

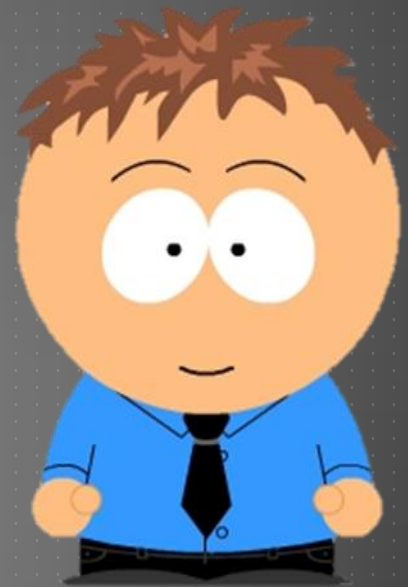
2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$



SOLUTIONS

2.

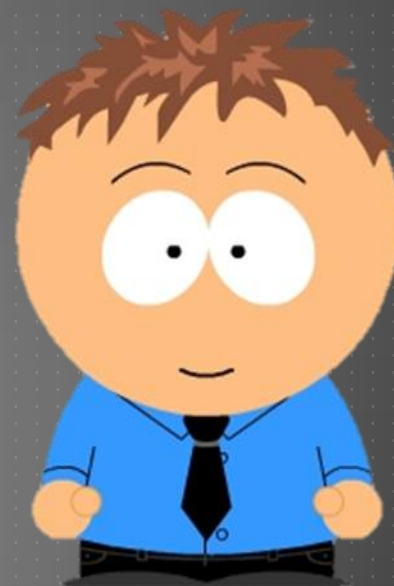
$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

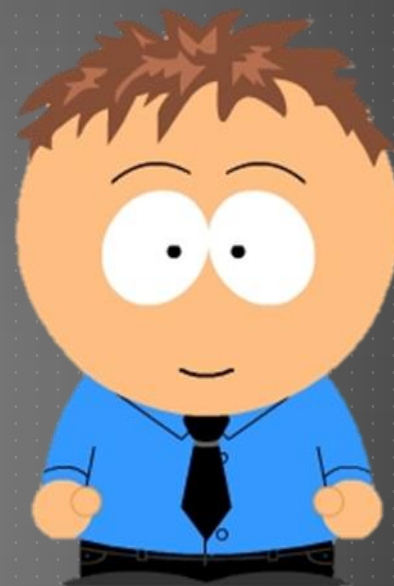
Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2$$

$$\frac{du}{-3x^2} = dx$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

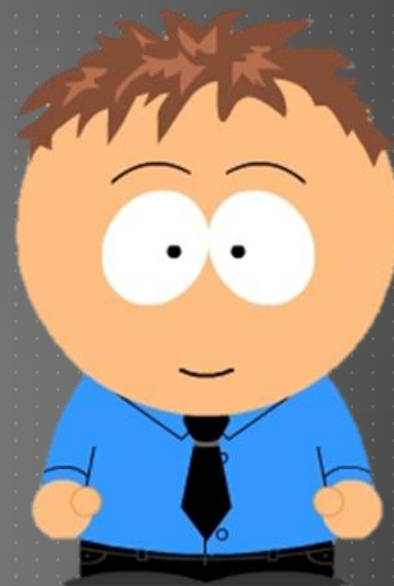
Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2 \quad \frac{du}{-3x^2} = dx$$

$$\int 5x^2 u^{\frac{1}{2}} \frac{du}{-3x^2}$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

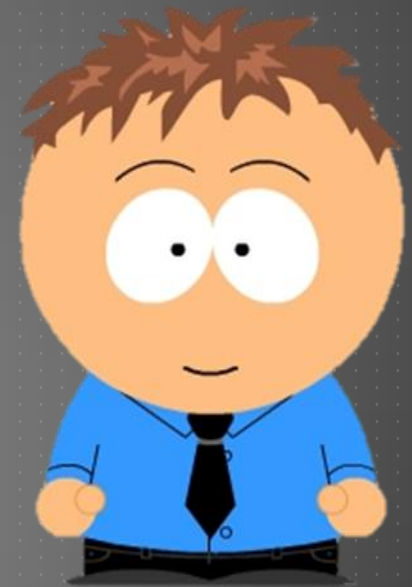
Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2 \quad \frac{du}{-3x^2} = dx$$

$$\int 5x^2 u^{\frac{1}{2}} \frac{du}{-3x^2} = \int -\frac{5}{3} u^{\frac{1}{2}} du$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $\int_0^1 g(x) dx$

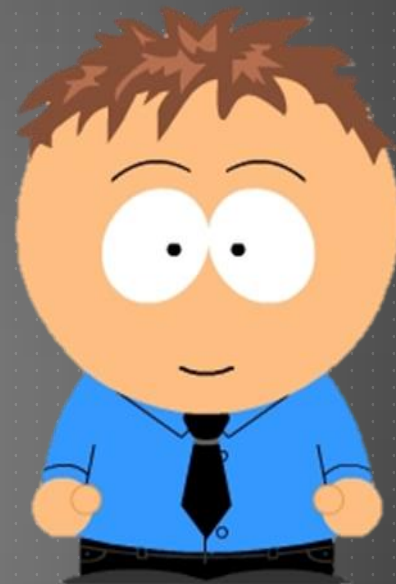
$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2 \quad \frac{du}{-3x^2} = dx$$

$$\int 5x^2 u^{\frac{1}{2}} \frac{du}{-3x^2} = \int -\frac{5}{3} u^{\frac{1}{2}} du = -\frac{10}{9} u^{\frac{3}{2}}$$

Remember that when integrating between limits we **don't need to include '+c'**



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

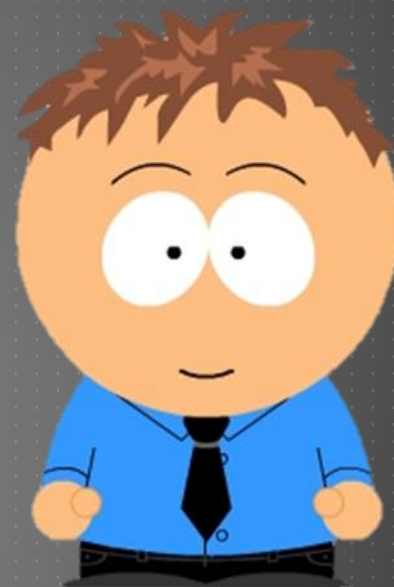
Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2 \quad \frac{du}{-3x^2} = dx$$

$$\int 5x^2 u^{\frac{1}{2}} \frac{du}{-3x^2} = \int -\frac{5}{3} u^{\frac{1}{2}} du = -\frac{10}{9} u^{\frac{3}{2}}$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $\int_0^1 g(x) dx$

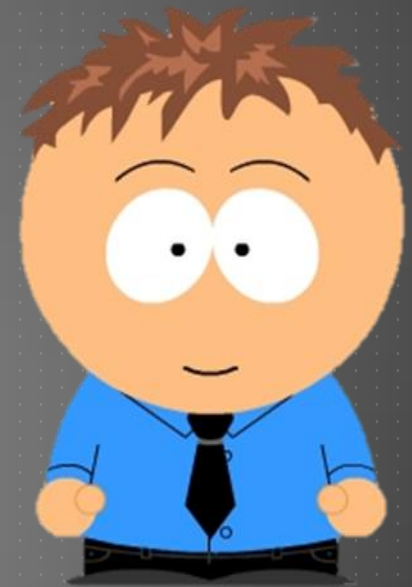
$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2 \quad \frac{du}{-3x^2} = dx$$

$$\int 5x^2 u^{\frac{1}{2}} \frac{du}{-3x^2} = \int -\frac{5}{3} u^{\frac{1}{2}} du = -\frac{10}{9} u^{\frac{3}{2}}$$

$$\left[-\frac{10}{9} (1-x^3)^{\frac{3}{2}} \right]_0^1$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $\int_0^1 g(x) dx$

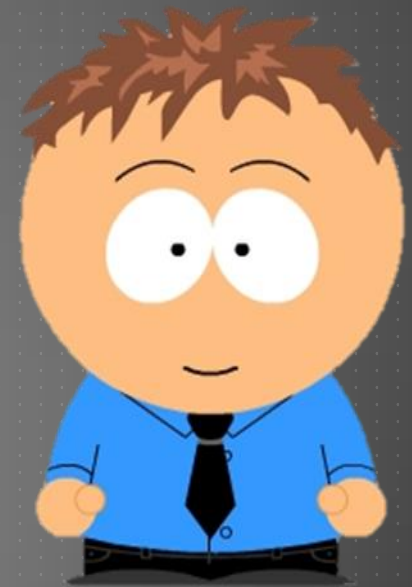
$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2 \quad \frac{du}{-3x^2} = dx$$

$$\int 5x^2 u^{\frac{1}{2}} \frac{du}{-3x^2} = \int -\frac{5}{3} u^{\frac{1}{2}} du = -\frac{10}{9} u^{\frac{3}{2}}$$

$$\left[-\frac{10}{9} (1-x^3)^{\frac{3}{2}} \right]_0^1 = 0 - -\frac{10}{9}$$



SOLUTIONS

2.

$$g(x) = 5x^2\sqrt{1-x^3}$$

Find $\int_0^1 g(x) dx$

$$\int_0^1 5x^2\sqrt{1-x^3} dx$$

$$u = 1 - x^3$$

$$\frac{du}{dx} = -3x^2 \quad \frac{du}{-3x^2} = dx$$

$$\int 5x^2 u^{\frac{1}{2}} \frac{du}{-3x^2} = \int -\frac{5}{3} u^{\frac{1}{2}} du = -\frac{10}{9} u^{\frac{3}{2}}$$

$$\left[-\frac{10}{9} (1-x^3)^{\frac{3}{2}} \right]_0^1 = 0 - \left(-\frac{10}{9} \right) = \frac{10}{9}$$

