DIFFERENTIATION AND INTEGRATION PART 4

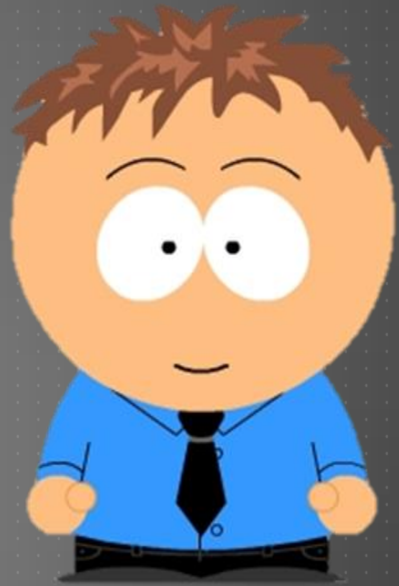
Mr C’s IB Standard Notes

In this PDF you can find the following:

1. Kinematics
2. Worked Example

KINEMATICS

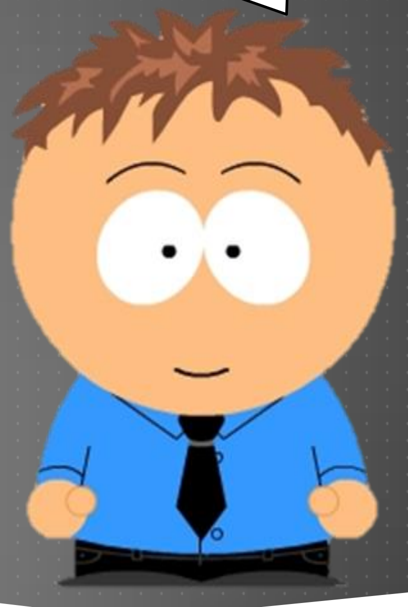
Working with displacement, velocity and acceleration



KINEMATICS

Working with displacement, velocity and acceleration

One very useful application of calculus is in the topic of *kinematics*

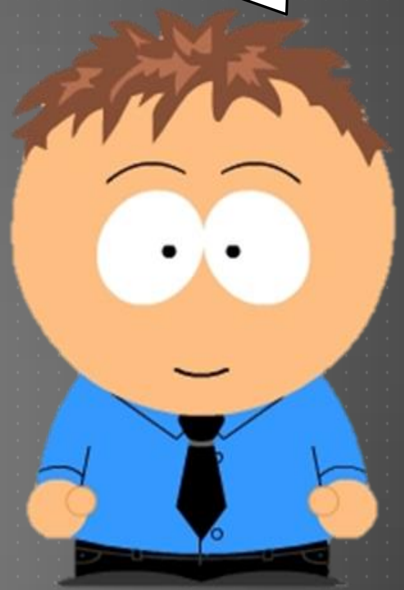


KINEMATICS

Working with displacement, velocity and acceleration

Most people know the relationship

$$speed = \frac{distance}{time}$$

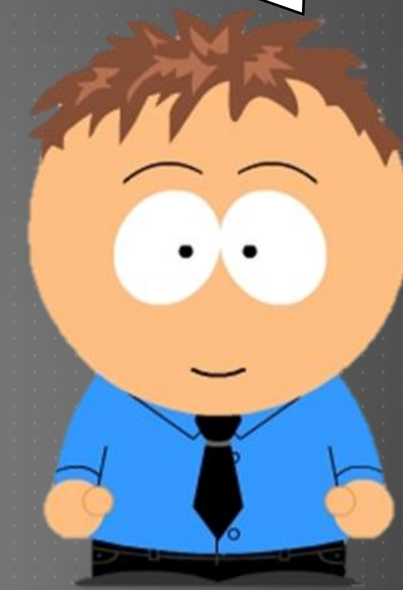


KINEMATICS

<u>Working with displacement, velocity and acceleration</u>

When working with vector quantities this becomes

$$velocity = \frac{displacement}{time}$$



KINEMATICS

Working with displacement, velocity and acceleration

$Displacement = s,\ velocity = v, Acceleration = a$

$Time = t$

KINEMATICS

<u>Working with displacement, velocity and acceleration</u>

$Displacement = s,\ velocity = v, Acceleration = a$

$Time = t$

$$v = \frac{s}{t}$$

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KINEMATICS

Working with displacement, velocity and acceleration

$Displacement = s,\ velocity = v, Acceleration = a$

$Time = t$

$$v = \frac{s}{t}$$

More precisely it can be stated as

$$velocity = \frac{change\ in\ distance}{change\ in\ time}$$

KINEMATICS

<u>Working with displacement, velocity and acceleration</u>

$Displacement = s,\ velocity = v, Acceleration = a$

$Time = t$

$$v = \frac{\Delta s}{\Delta t}$$

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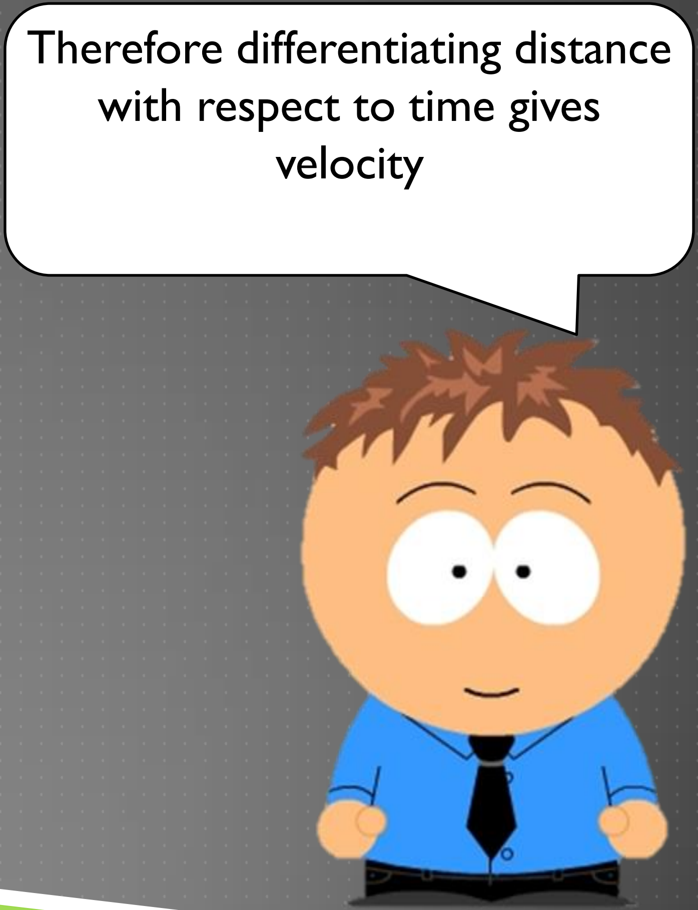


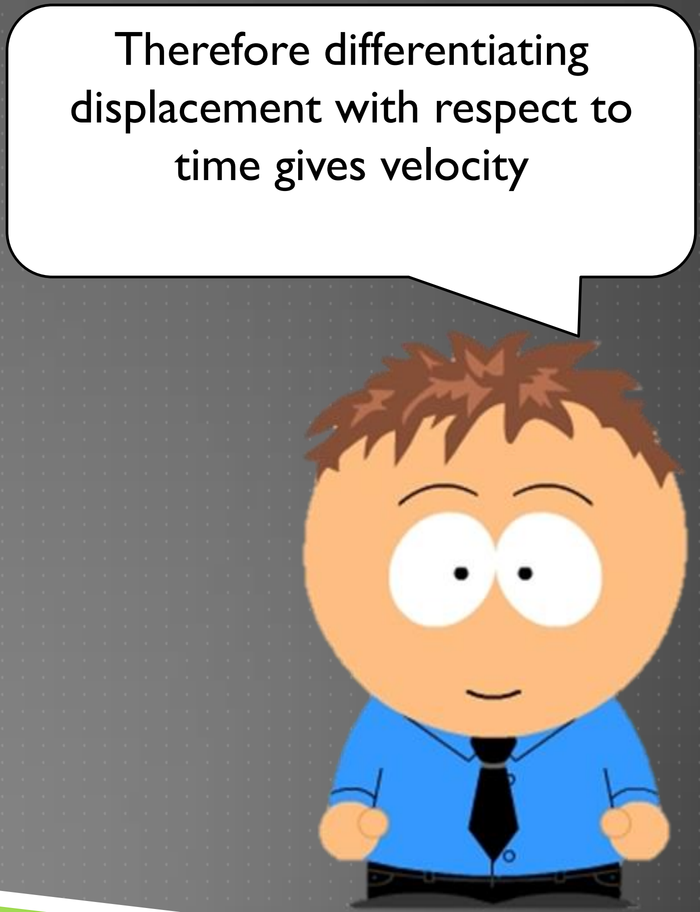
KINEMATICS

<u>Working with displacement, velocity and acceleration</u>

$Displacement = s,\ velocity = v, Acceleration = a$

$Time = t$

$$v = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$



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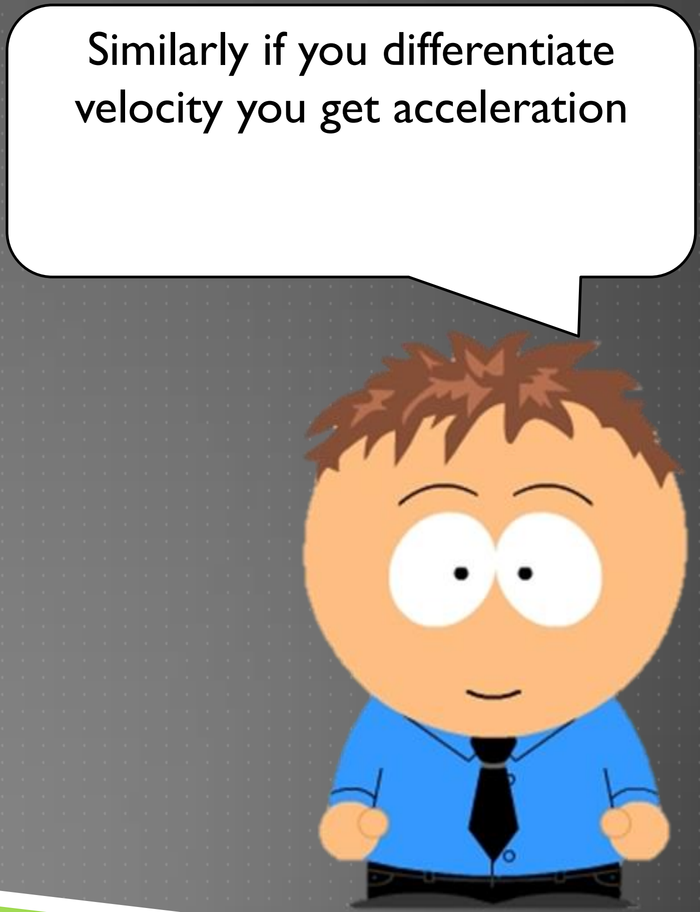

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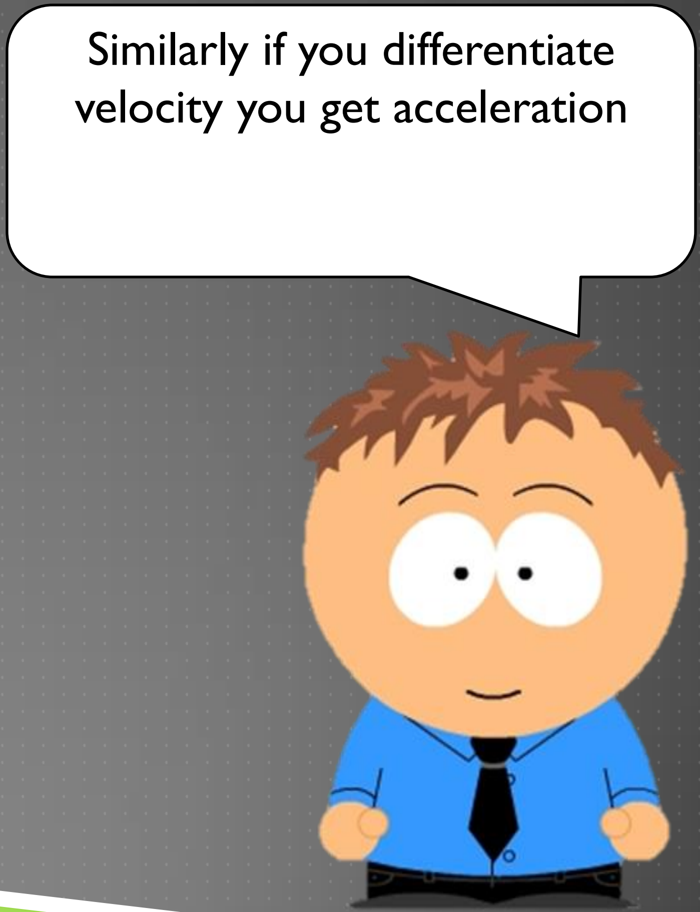

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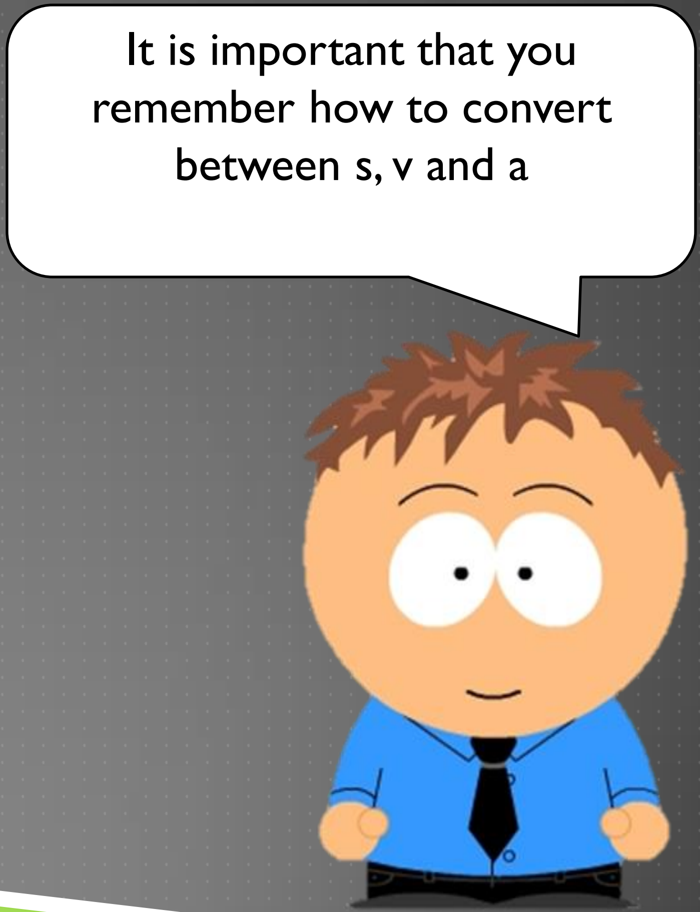

KINEMATICS

<u>Working with displacement, velocity and acceleration</u>

$Displacement = s,\ velocity = v, Acceleration = a$

$Time = t$

differentiate

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$$v = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

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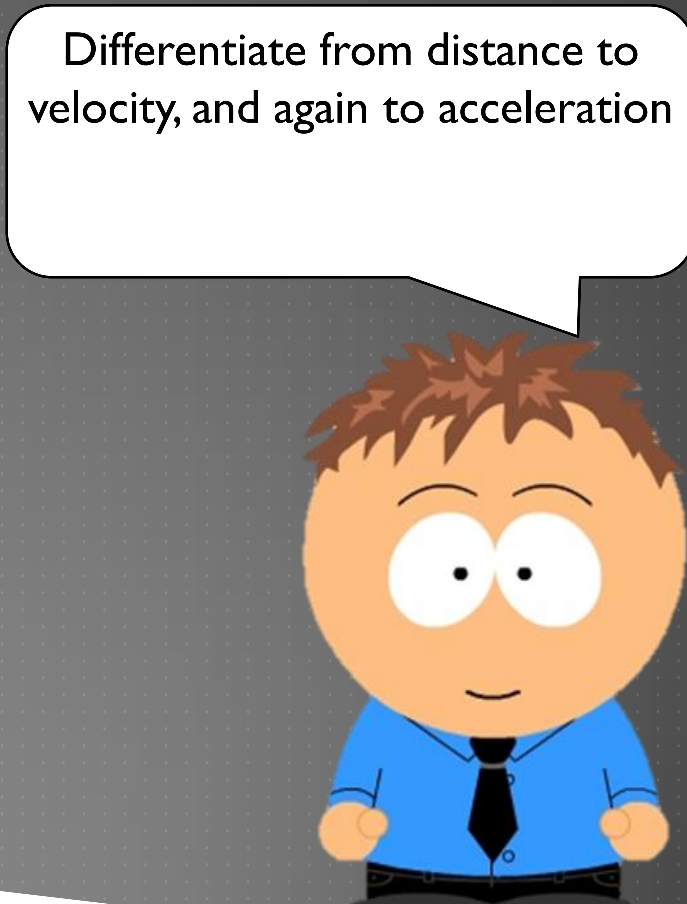

KINEMATICS

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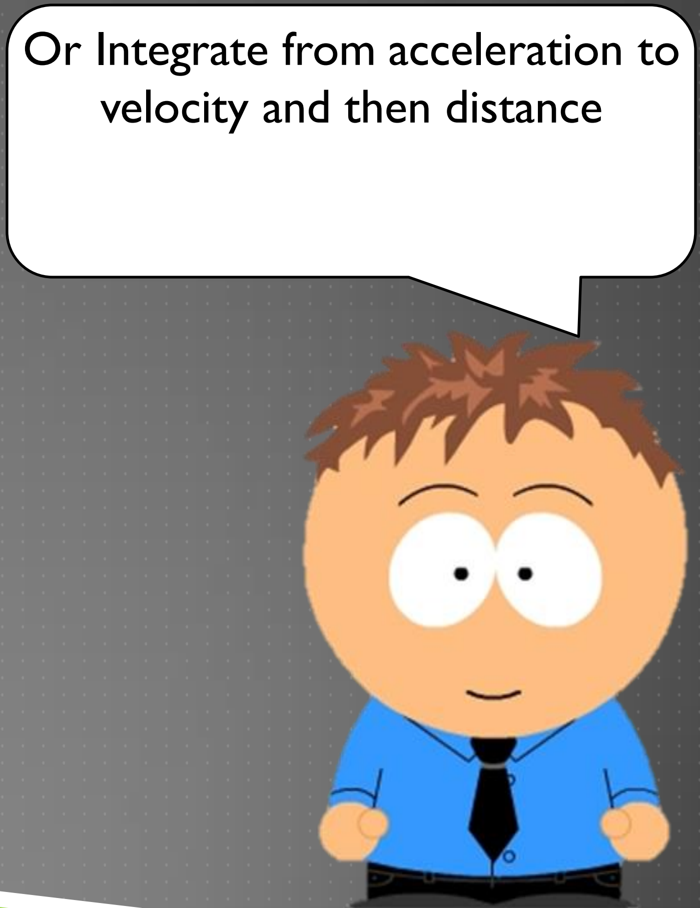
integrate



EXAMPLE

1.

Amy does a bungee jump from a platform 50m above a river. Let h be her height above the river, in metres, at a time t seconds after jumping.
Her velocity is given by $v = 2t^2 - 10t$

a) What is the initial acceleration Amy experiences?

b) At what time is Amy's velocity zero?

c) How close to the river does Amy get?

SOLUTION

Amy does a bungee jump from a platform 50m above a river. Let h be her height above the river, in metres, at a time t seconds after jumping. Her velocity is given by $v = 2t^2 - 10t$

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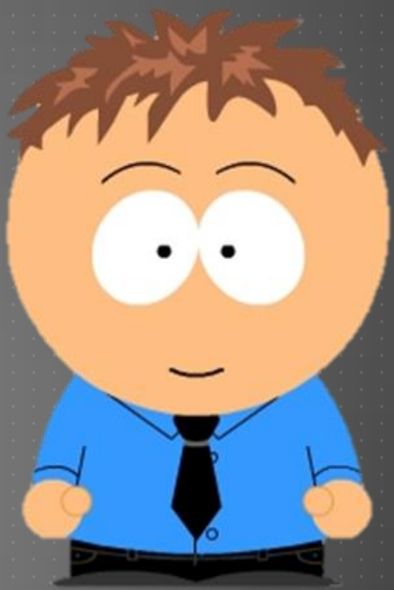
We have a formula for *velocity*, but are asked to find *acceleration*. We must therefore differentiate



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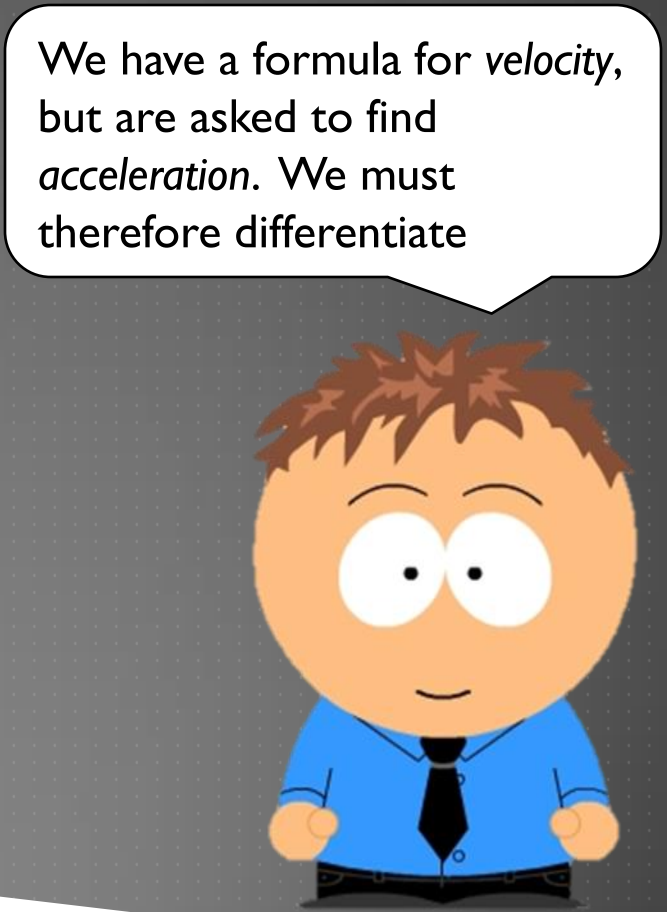

SOLUTIONS

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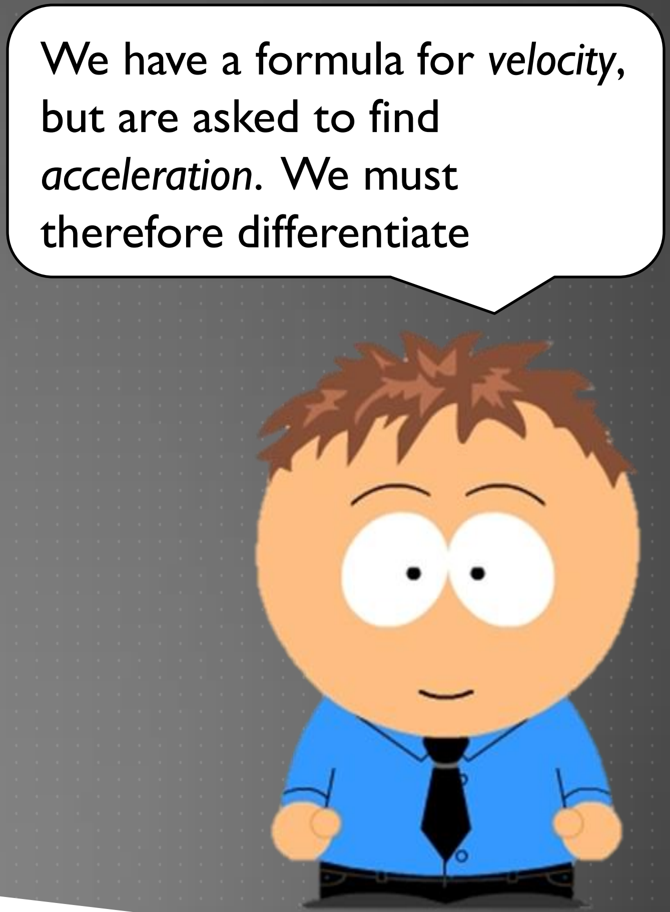
$$v = 2t^2 - 10t$$

$$a = \frac{dv}{dt} = 4t - 10$$



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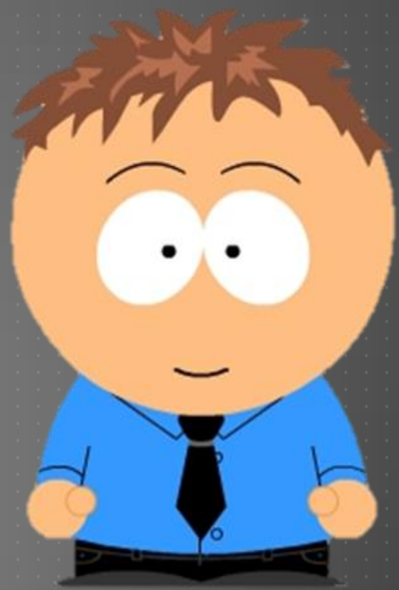
The **initial acceleration** occurs at the start, i.e. when **t=0**



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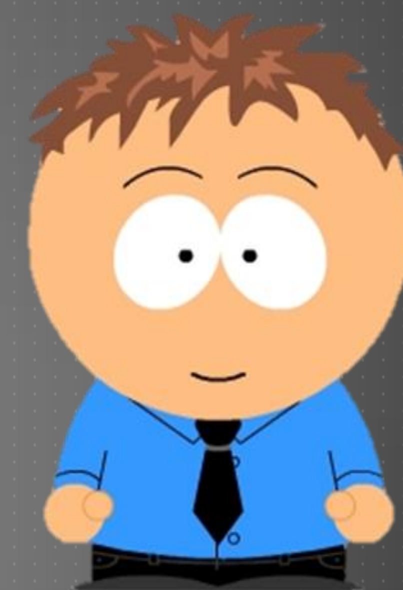

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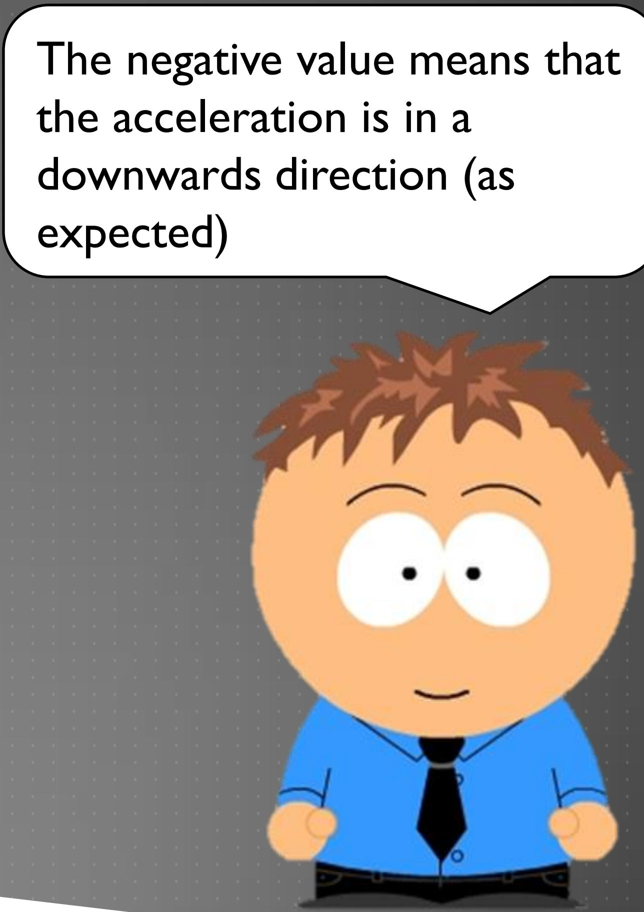

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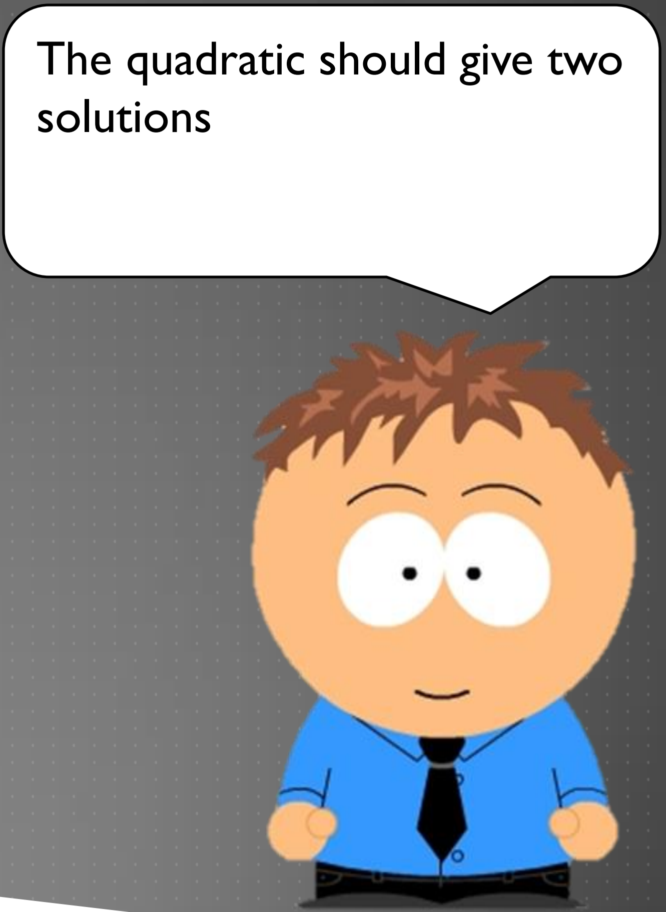
$$\boldsymbol{t = 0 \text{ or } t = 5}$$



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Amy does a bungee jump from a platform 50m above a river. Let h be her height above the river, in metres, at a time t seconds after jumping. Her velocity is given by $v = 2t^2 - 10t$

c) How close to the river does Amy get?

$v = 2t^2 - 10t$ $\quad$ $v = 0 \; when \; t = 0 \; or \; t = 5$

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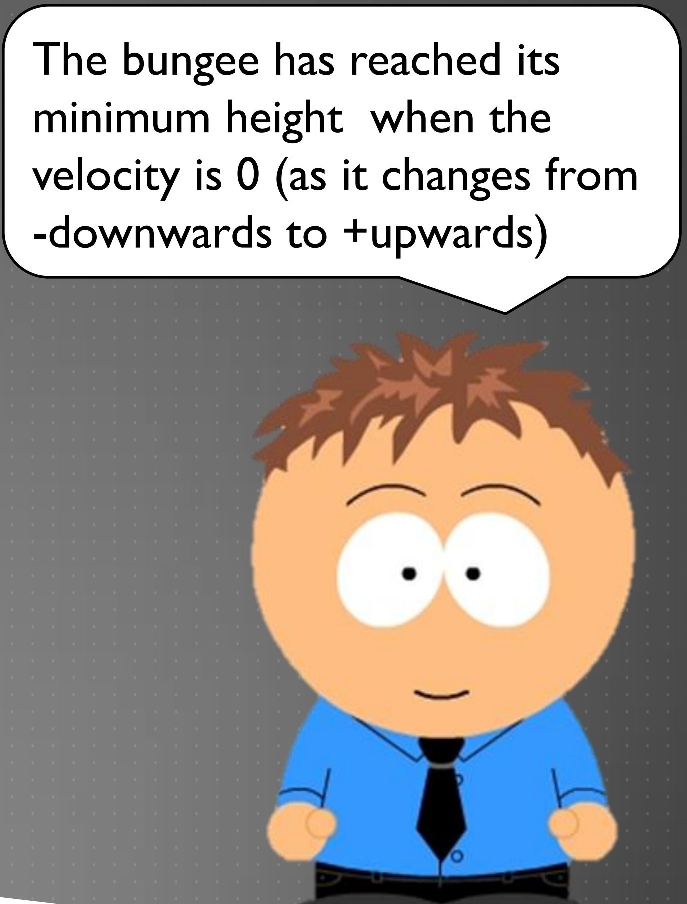

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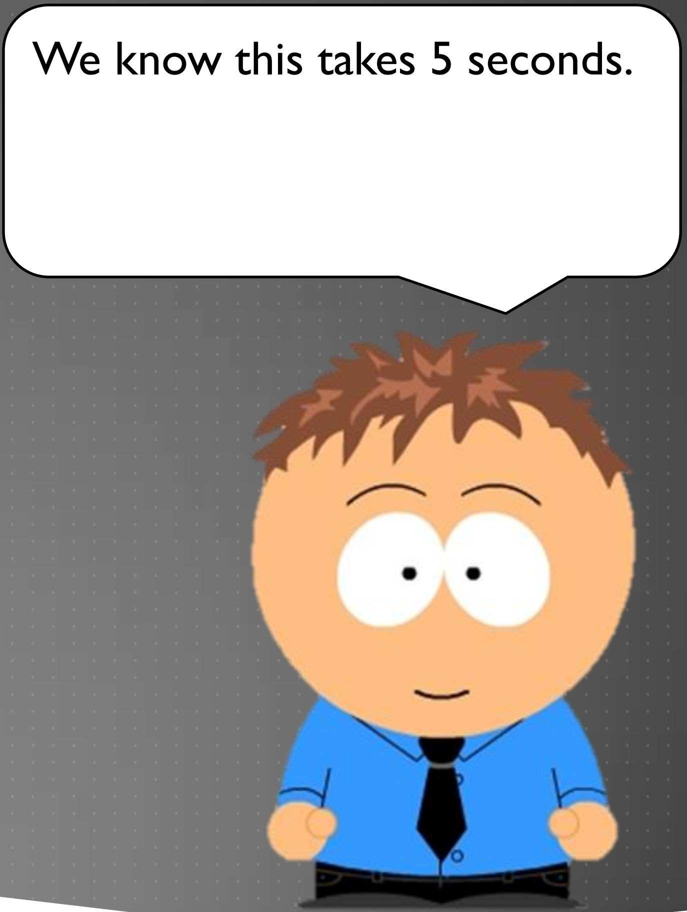

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So we must calculate how far is travelled in those 5 seconds

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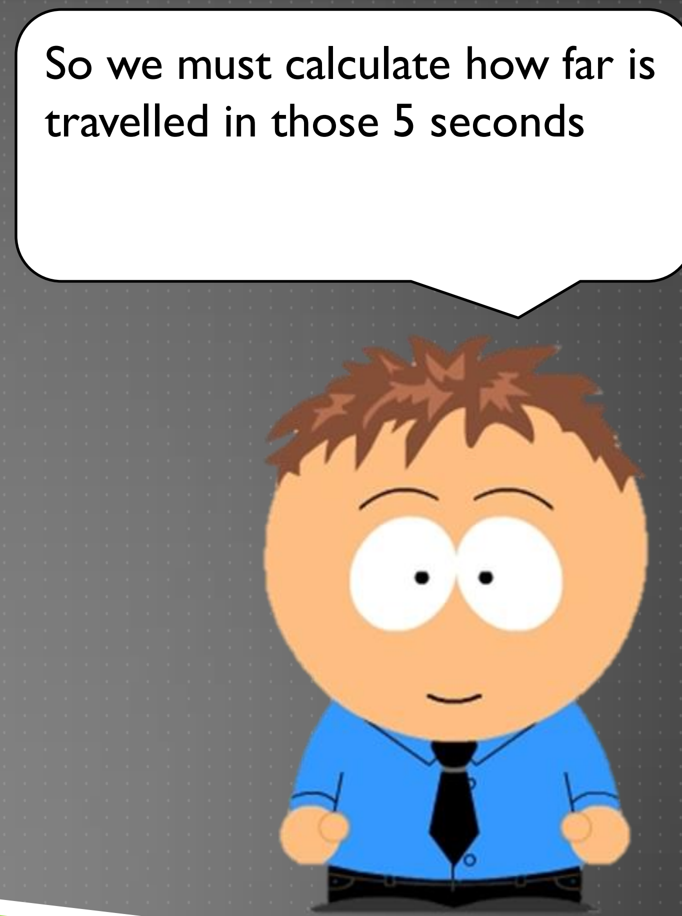
$s = \int v \, dt$



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$\int_0^5 2t^2 - 10t \; dt$

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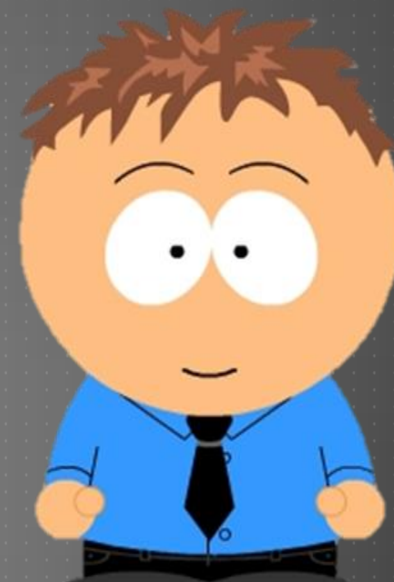

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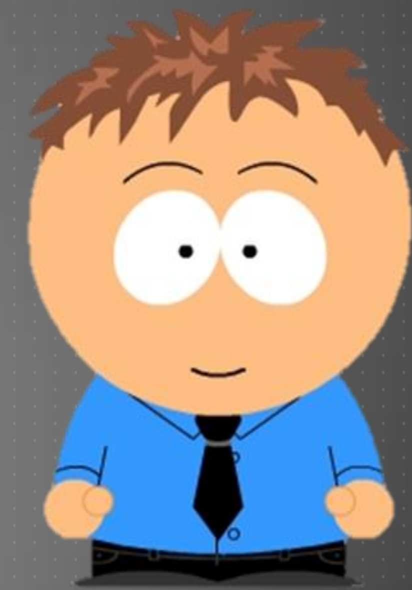
$\int_0^5 2t^2 - 10t \, dt \qquad = \left[\frac{2}{3}t^3 - 5t^2\right]_0^5$

So we must calculate how far is travelled in those 5 seconds



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$= -41\frac{2}{3}$ m

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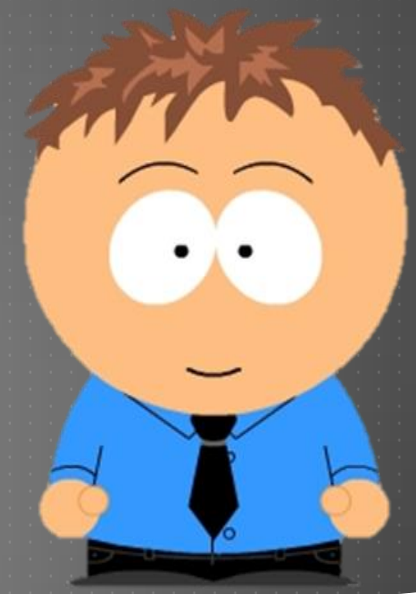

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$$= -41\frac{2}{3} \text{ m}$$

$50 - 41\frac{2}{3} = 8\frac{1}{3}$ m from the river